Tubular Steel Truss Design Using Semi-rigid Joints

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ABSTRACT

Despite the fact that semi-rigid nature of steel structures joints is well-known for several decades the truss design is constantly being carried out with ideally hinged joints in the finite element models. In some recent studies the stiffness of welded tubular truss joints have been defined as well as the member buckling lengths in the trusses. The aim of this paper is to see the effects of applying the stiffness and buckling length knowledge in truss design procedure. By an example case it is shown that joints can be treated almost ideally rigid and that in brace design shorter values for buckling length could be used than those proposed by Eurocode standards.

INTRODUCTION

Steel frames including columns using semi-rigid joints have proven efficient solutions. Considerable cost savings have been shown for sway frames when using semi-rigid joints instead of stiff joints (Simoes, 1996). Cost savings have been shown when using semi-rigid joints instead of hinged joints in non-sway frames as well (Bzdawka, 2012). Tubular steel trusses with welded joints are designed following standards, such as Eurocodes, supposing hinges to the joints between braces and chords. This is due to fact that the rotational stiffness of these welded joints has not been known until recently. Snijder et al (2011) studied the rotational stiffness of tubular welded joints using a comprehensive 3D finite element analysis. Corresponding buckling lengths for the members of the truss were defined by eigenmode analysis.

The goal of present paper is to show the effects of different local joint analysis models to resistance and other requirements for the members and the joints in normal conditions. The requirements originate from Eurocode standards. A simple one span symmetric roof truss made of cold-formed square tubes is considered. When performing all checks required in Eurocodes the starting point should be the geometrical model of the truss. This enables definitions of eccentricity elements at joints and checking of other geometrical constraints, as well. Three local joint analysis models are considered: Braces connected to chords with hinges and with eccentricity elements, same but using ideally rigid joints instead of hinges and finally the most realistic semi-rigid joints including control of buckling lengths following the Dutch study (Snijder et al, 2011). Global structural models are linear elastic plane FEMs with beam elements. Member and joint resistances using these three theories are given in the paper and possibilities to reduce the member buckling lengths are demonstrated using semi-rigid joints.

FROM GEOMETRY MODEL TO FINITE ELEMENT MODEL

In tubular truss design the imaginary extensions of brace members do not typically meet at same point eg. there is eccentricity in the joint. Three examples are seen in Figure 1. In the option a) the eccentricity is not taken into account and the brace elements are connected to the chord in the middle of the gap. In the option b) eccentricity element is used in the joint to connect braces with chords. The option c) uses eccentricity element for each brace connected to chord. The latter two take eccentricity into account when considering chord moment.

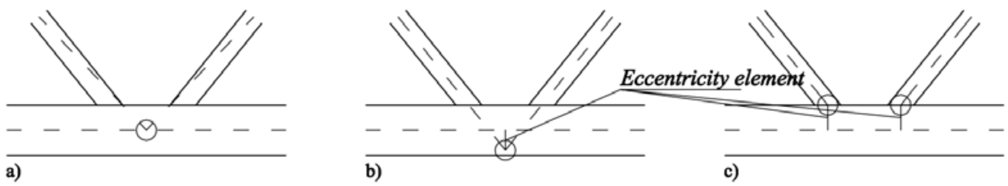

Figure 1. Different types of finite element models derived from the geometrical model.

The hinge can be also replaced with a rotational spring to model the semi-rigid nature of the joint. To consider the appropriate stiffness of the spring a quantity called *fixity factor* is introduced. The fixity factor is defined as ratio of angles seen Figure 2.

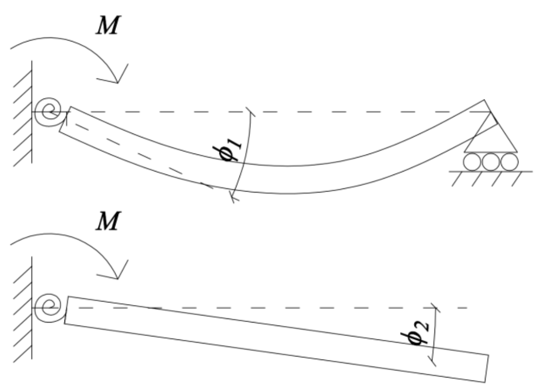

Figure 2. Definition of fixity factor.

It can be written as equation (Monforton and Wu, 1963)

$$\alpha = \frac{\phi_2}{\phi_1} = \frac{\dfrac{M}{k + \dfrac{EI}{L}}}{\dfrac{M}{k}} = \frac{1}{1 + \dfrac{3EI}{kL}}$$

or if fixity factor is known, the respective rotational stiffness can be solved, thus

$$k = \frac{3EI}{L}\frac{\alpha}{1 - \alpha}$$

Fixity factor can get values $\alpha \in [0,1]$ where 0 means ideally hinged and 1 means ideally rigid connection. Eurocode 1993-1-8, on the other hand, gives values for the limits of ideally rigid or hinged joints. A joint can treated ideally rigid if the stiffness satisfies

$$k \geq \frac{25EI}{L}$$

or by using fixity factor

$$\alpha \geq \frac{25}{28} = 0.893$$

Also, if stiffness satisfies

$$k \leq \frac{EI}{2L} \quad \text{or} \quad \alpha \geq \frac{1}{7} = 0.143$$

the joint can be treated ideally hinged.

STRUCTURAL ANALYSIS AND TRUSS EVALUATION

In this work structure is evaluated following Eurocode standards 1993-1-1, (CEN, 2005) and 1993-1-8 (CEN, 2005). Finite element method using *Euler-Bernoulli* beam element with rotational springs proposed by Jalkanen (2004) is utilized. As the geometry is locked several types of beam finite element models can be used.

The bending moment found in braces of models where brace joints are either semi-rigid or rigid was taken into account in joint design by replacing design axial force N_{Ed} with expression proposed by Wardenier (1982):

$$\bar{N}_{Ed} = N_{Ed} + \frac{|M_{y,Ed}|}{W_{y,el}} \cdot A$$

Members are checked against shear, axial force, bending and the combination of these two. The joints are checked as K-joints or when certain requirements are not fulfilled, as two separate T-joints.

The rotational stiffness of a K-joint was found dependent of two parameters also used in Eurocode nomenclature, namely γ and β. They are defined by equations

$$\gamma = \frac{b_0}{2t_0}$$

and

$$\beta = \frac{b_i}{b_0}$$

where b_0 is the width of the chord, t_0 wall thickness of the chord, and b_i width of the brace. The stiffness was defined by finite element method at points shown in Table 1. The values used in this work are acquired by using linear interpolation.

The ratio of buckling length and member system length for braces is given by equation

$$K = \frac{L_{cr}}{L_{sys}} = (A + B\gamma)\left(\beta \frac{b_1}{L}\right)^C + D$$

in which parameters *A*, *B*, *C* and *D* are given in Table 2. For chords, the buckling length ratio is calculated as

$$K = A + B\beta \leq 0.9$$

In Eurocodes, $K = 0.75$ is used for braces and $K = 0.9$ for compressed chords.

Table 1. Joint stiffness sample points by Boel, 2010.

γ	β	k [kNm/rad]
15.87	0.25	82
	0.4	188
	0.5	356
	0.6	712
	0.75	2766
	0.9	12295
	1	33576
10	0.25	292
	0.4	628
	0.5	1133
	0.6	2142
	0.75	6888
	0.9	24808
	1	53409
6.25	0.25	953
	0.4	2145
	0.5	3681
	0.6	6547
	0.75	17718
	0.9	52422

Table 2. Values of parameters

	A	B	C	D
Brace	1.05	0.025	0.14	0
Chord	1.25	-0.6	-	-

RESULTS WITH DIFFERENT APPROACHES

Let us first consider a Warren type truss with equal node spacing and span of 36 metres as seen in Figure 3. Material is S420 steel. Applying distributed force of $q_d = 24.9$ kN/m, utilization ratios against bending moment, axial force and combined interaction of these forces can be calculated as well as the resistance checks for the joints.

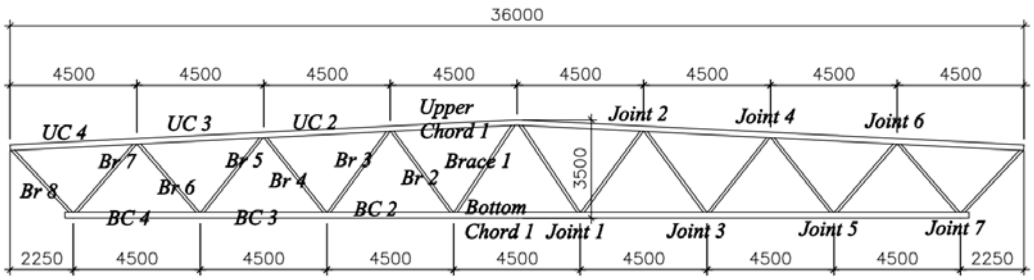

Figure 3. Example Warren type truss.

The chosen profiles were selected manually to give the lightest possible structural weight without violating any constraint found in Eurocodes using hinged brace elements. In Table 3 these profiles are seen as well as the joint stiffnesses and respective fixity factors of the joints at both ends. The braces are numbered starting from the middle of the span as in Figure 3. It should be noted that range of stiffness is pretty large but fixity factors have pretty high values, over half of the values implying the joints could be treated ideally rigid when following Eurocode 1993-1-8.

Table 3. Member profiles and end stiffnesses

	Chords		Braces							
	Upper	Bottom	1	2	3	4	5	6	7	8
Profile	180x10	160x6	60x3	70x3	90x3	70x3	110x4	80x3	140x5	140x5
k_1 [kNm/rad]	-	-	344	991	1090	991	3207	1379	15510	13924
k_2 [kNm/rad]	-	-	782	496	1813	496	3795	692	13924	15510
α_1	-	-	0.86	0.92	0.84	0.91	0.86	0.90	0.92	0.91
α_2	-	-	0.94	0.84	0.90	0.84	0.88	0.82	0.91	0.91

It was found that regardless of stiffness assumption the axial forces were more or less the same. This was also the case without eccentricity elements. On the other hand, the bending moments differ substantially. The values can be seen in Table 4. Element numbering follows Figure 3.

The buckling length factors and resistance utility ratios were calculated against axial force, bending and the interaction of these two for elements are seen in Table 5. With this truss, the chord elements take the buckling length value proposed by the Eurocode as well even when treated as semi-rigid. The braces, on the other hand, can be treated shorter which can be beneficial. In this table only plane case was evaluated.

Table 4. Member stress resultants with different analysis models

	Hinged		Semi-rigid		Rigid	
Element	*M* [kNm]	*N* [kN]	*M* [kNm]	*N [kN]*	*M* [kNm]	*N* [kN]
Br 1	0	4.6	0.05	4.6	0.06	4.6
Br 2	0	-3.7	0.05	-3.9	0.06	-4.0
Br 3	0	-138	0.21	-138	0.25	-137
Br 4	0	140	0.07	141	0.07	142
Br 5	0	-295	0.54	-296	0.62	-296
Br 6	0	322	0.01	317	0.02	317
Br 7	0	-514	0.08	-509	0.09	-509
Br 8	0	524	14.42	527	15.35	528
UC 1	40	-1228	40	-1228	40	-1228
UC 2	35	-1150	35	-1151	35	-1151
UC 3	43	-885	43	-884	43	-884
UC 4	57	-352	57	-357	57	-358
BC 1	2.1	1224	2.3	1224	2.3	1224
BC 2	5.9	1229	5.0	1229	4.9	1229
BC 3	2.6	1063	2.4	1062	2.6	1062
BC 4	38	676	24	679	23	679

The joint resistance checks were also performed using the different models. Every K-joint was evaluated checking four failure modes: 1. chord face yield 2. chord shear 3. chord punch shear 4. brace failure. The utility ratio of critical failure mode can be seen in Table 6. Joint numbering follows Figure 3.

In the joint resistance utilitities the values are similar regardless of the analysis model in all other joints but number seven. In that joint, the rather large moment contributes to a large axial force $\overline{N}_{Ed}$ which causes the joint to fail.

DISCUSSION AND CONCLUSION

In this study the semi-rigid nature of joints of tubular trusses was introduced to resistance evaluation procedure. The effect of joint rotational stiffness does not seem as significant as with frames. In the example truss the stiffnesses of the joints are either ideally rigid or close to it. This will cause bending moments to braces as well. The moments are pretty small except the outermost brace in which the bending moment becomes significant. Axial forces and chord moments are similar with hinged, semi-rigid and rigid models only exception being outermost element at the bottom chord in which moment is reduced in semi-rigid and rigid models.

The buckling lengths in most braces are reduced but in some braces longer than proposed by Eurocode. The most important members regarding to structural mass and cost are the chords. If brace member sizes are chosen properly chord buckling length could be reduced resulting in savings. As this phenomenon is very complex it cannot be verified without rigorous computational effort. Therefore, only very careful conclusions can be made and further research is needed. In addition it should be noted that reference used in this study had certain limitations. It does not give buckling length with different stiffnesses at the member ends. In this work, average approximation was used.

Table 5. Buckling length factors and utility ratios using different analysis models

	Hinged				Semi-rigid				Rigid			
Element	*K*	*N*	*M*	*M* + *N*	*K*	*N*	*M*	*M* + *N*	*K*	*N*	*M*	*M* +*N*
Br 1	0.75	0.02	0.00	0.02	0.64	0.02	0.01	0.02	0.75	0.02	0.01	0.03
Br 2	0.75	0.04	0.00	0.04	0.67	0.03	0.01	0.04	0.75	0.04	0.01	0.04
Br 3	0.75	0.71	0.00	0.71	0.72	0.67	0.02	0.69	0.75	0.71	0.02	0.73
Br 4	0.75	0.43	0.00	0.43	0.67	0.43	0.01	0.44	0.75	0.43	0.01	0.44
Br 5	0.75	0.72	0.00	0.72	0.77	0.73	0.02	0.75	0.75	0.72	0.02	0.74
Br 6	0.75	0.85	0.00	0.85	0.70	0.84	0.00	0.84	0.75	0.84	0.00	0.84
Br 7	0.75	0.63	0.00	0.63	0.82	0.67	0.00	0.67	0.75	0.63	0.00	0.63
Br 8	0.75	0.47	0.00	0.47	0.83	0.48	0.26	0.74	0.75	0.48	0.28	0.75
UC 1	0.9	0.72	0.23	0.93	0.90	0.72	0.23	0.93	0.90	0.72	0.23	0.93
UC 2	0.9	0.67	0.20	0.90	0.90	0.67	0.21	0.90	0.90	0.67	0.21	0.90
UC 3	0.9	0.52	0.25	0.70	0.90	0.52	0.25	0.70	0.90	0.52	0.25	0.70
UC 4	0.9	0.21	0.34	0.44	0.78	0.19	0.34	0.42	0.90	0.21	0.34	0.45
BC 1	0.9	0.81	0.02	0.83	0.90	0.81	0.03	0.84	0.90	0.81	0.03	0.84
BC 2	0.9	0.81	0.07	0.88	0.90	0.81	0.06	0.87	0.90	0.81	0.06	0.87
BC 3	0.9	0.70	0.03	0.73	0.90	0.70	0.03	0.73	0.90	0.70	0.03	0.73
BC 4	0.9	0.45	0.44	0.89	0.84	0.45	0.28	0.72	0.90	0.45	0.26	0.71

Table 6. Buckling length factors and utility ratios using different analysis models

	K-Joint						
	1	2	3	4	5	6	7
Rigid	0.90	0.51	0.92	0.59	0.99	0.92	0.87
Semi-rigid	0.90	0.50	0.92	0.60	0.97	0.91	2.00
Hinged	0.90	0.50	0.92	0.61	0.97	0.91	2.00

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