

An Interactive Virtual Environment to Improve Undergraduate Students' Competence in Surveying Mathematics

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ABSTRACT

In this paper we describe the development of an interactive virtual environment whose objective is to improve undergraduate students' representational fluency in surveying mathematics. One problem with teaching surveying is a general decline in the math skills of students taking surveying courses, and a decline in the students' ability to transfer these concepts into practice in the context of surveying. In the paper we explain how an interactive virtual environment that presents multiple interconnected representations of math and geometry concepts, e.g., dynamic visual objects, symbolic or numeric expressions, and written and verbal explanations simultaneously on the computer screen, can help students acquire the math and geometry skills necessary to perform surveying tasks such as chaining and triangulations and coordinate calculations.

INTRODUCTION

Surveying is the science of studying the 3D shapes of the earth's curvature and surveying concepts are founded in geometry and vectors principles. In the context of construction layout, angles, distances and elevations are used to set up the building footprint at the correct location, establish level elevations and plumb vertical surfaces. Traditionally, in a surveying course, instructors teach the theoretical foundation of surveying, which includes math, trigonometry, geometry and physics concepts. These concepts are often explained through examples from textbooks and illustrations on the chalkboard.

One major problem in teaching surveying is what R. Elgin calls "The Demise of Basic Surveying Mathematics" (The American Surveyor, May 2007). Elgin reports "...a distinct decline in the math skills of students taking surveying courses...", and a decline in the students' ability to transfer these concepts into practice in the context of surveying. In other words, students not only lack basic math knowledge, but also 'representational fluency', i.e. the ability to reason with and among multiple interconnected representations (symbolic, textual, pictorial, concrete).

One of the main objectives of the work described in the paper is to improve students' representational fluency in surveying mathematics through the use of interactive virtual environments. An appealing characteristic of virtual environments is their interactivity and capability to present multiple representations at the same time on the computer screen (e.g., dynamic visual objects, symbolic or numeric expressions, and written explanations simultaneously). The work reported in the paper aims to extend this research to the undergraduate classroom.

BACKGROUND

Representational Fluency in Mathematics. An important aspect of mathematical competence is the ability to reason with and among multiple representations; this skill is called representational fluency. The importance of this skill has increasingly been recognized by the mathematics education community and is also being recognized in other fields where the use and application of mathematical concepts is essential. The mathematics education community has acknowledged that there has historically been a preoccupation with numbers as predominate means of teaching about mathematical constructs.

Kaput (1989) suggested that mathematical 'meaning making' is built upon the ability to go beyond knowing how to work with mathematical concepts in any particular representational form, i.e., symbols, graphs, tables, verbal expressions, instances, visualizations, etc., to the ability to translate within and among representations. In specifically demonstrating the need for representational fluency, Knuth (2000) found that algebra students who are familiar with equations and graphs do not readily connect graphical representations, e.g., the Cartesian coordinate system, to their knowledge of equations; these students failed to translate the mathematical concept from one form to the next. This is what some would call "brittle knowledge" meaning that it quickly breaks. The benefits of representational fluency then is that mathematical concepts are learned at a deeper and more enduring level, where deep understanding promotes subsequent successful learning (transfer to learning) as well as in assists in the transfer of knowledge to practical application. The importance of representational fluency extends beyond the classroom into the workplace, where it is seen as an essential skill, mediating data driven decision making, mathematical and scientific modeling, the interpretation and explanation of complex systems, and the use of increasingly advanced technology (Dark, 2003; English, 2007).

A small but growing body of classroom research has begun to emerge on uses of 'virtual manipulatives' to improve math competence. An appealing characteristic of these technological tools is their interactivity and capability to present multiple representations at the same time on the computer screen. This enables the user to have parallel views of both visual and written expressions of the same mathematical concept or process, which is necessary for developing representational fluency.

Interactive Virtual Environments and Learning. The name "Virtual Learning Environment (VLE)" is used broadly and because of this broad usage, it can refer to a website with static web pages. At the other end of the continuum, the term VLE can

refer to a highly integrated system, including the associated infrastructure, that: a) substantively incorporates 3D virtual reality technology (i.e., an environment that represent objects, including manipulatives, with a high degree of fidelity), b) is immersive, meaning that educational interactions occur within the environment making the environment a “place”, c) is a designed information space in which the information is explicitly represented and the educational interactions occur so that students are not only active, but actors, i.e., they co-construct the information space, d) the VLE overlaps and is an extension of the physical environment, thereby possessing the potential to enrich classroom activities, and e) can integrate heterogeneous technologies and multiple pedagogical approaches (Dillenbourg, 2000).

Research on the educational benefits of interactive virtual learning environments is fairly recent and stems from the fields of computer graphics, cognitive psychology, visual cognition, and educational psychology. VR technology can present abstract concepts in concrete terms, offer the opportunity to manipulate concrete objects, and to bridge manipulatives with other representational forms. Technologies, such as VR, can be used to create interactive learning environments where learners can visualize concepts easily and receive feedback to build new knowledge and understanding (Hmelo 1998; Bransford 1999). Computer simulations have been shown to be an effective approach to improve student learning and have the potential to help students develop more accurate conceptions (Kangassalo 1994; Zietsman 1986; Gorsky 1992). Research shows that the use of simulation tools often reinforces learning and leads to performance improvements in a variety of disciplines. Therefore, recently, there has been significant progress in development of computer based tutorial systems in many different areas.

These early findings suggest the development of virtual learning environments has promise. In response, we are seeing the advancement of virtual learning environment projects in engineering and science. For example, Del Alamo (Mannix 2000), a professor of electrical engineering at MIT, created a web based microelectronics lab for his students in 1998. At Johns Hopkins University, Karweit (2005) has simulated various engineering and science laboratories on the web. At the University of Illinois Urbana-Champaign (UIUC), researchers have developed a virtual laboratory for earthquake engineering (SSTE 2005). While these projects are developing virtual learning environments, our team was unable to find publications with meaningful assessment and evaluation of student learning as well as usability for and by teachers.

More specific to this project, in the area of surveying, Kuo et al. (2007) have recently developed a virtual survey instrument (SimuSurvey) and studied the feasibility of introducing SimuSurvey in regular surveyor training courses. SimuSurvey teaches students only how to operate virtual surveying instruments, it does not reach back to the mathematical foundation and develop students' representational fluency nor does it challenge students with surveying exercises. At Leeds Metropolitan University, UK, Ellis et al. (2006) have developed an undergraduate VR surveying application. The interactive software includes 360-degree panoramic images of sites and makes use of QuickTime VR technology. The application was evaluated with 192 undergraduate students; findings suggest that the interactive tool complements traditional learning approaches, maintains student

interest, and reinforces understanding. At University of New Castle, UK, Mills and Barber (2008) have implemented a virtual surveying field course that includes both a virtual fieldtrip and a virtual interactive traverse learning tool (VITLT). The application was evaluated by several Geomatics students; all subjects highlighted the potential of VITLT to help the learning and understanding of a traverse.

THE VELs (VIRTUAL ENVIRONMENT FOR LEARNING SURVEYING): HOW IT CAN IMPROVE STUDENTS' REPRESENTATIONAL FLUENCY IN MATHEMATICS

The VELs is an interactive Virtual Environment designed to improve undergraduate students' learning of surveying concepts, practices and the underlying mathematics. It includes virtual terrains and surveying instruments that look, operate, and produce results comparable to the physical ones, and supports mathematical representational fluency by presenting the students with multiple interconnected representations of fundamental geometry and math concepts. It includes three educational modules: chaining, differential leveling, and triangulations and coordinate calculations. Detailed descriptions of each module can be found in Dib et al. (2014; 2011a; 2011b). In this paper we do not focus on a specific educational module, instead we discuss how the VELs can increase students' competence in surveying mathematics.

Below we give an example of a surveying exercise included in the VELs (measuring a horizontal distance with a steel tape – e.g. chaining) and we explain how this simple exercise can improve students' representational fluency in mathematics. This is one of the first exercises the students perform. Although this task might seem trivial at a first look, it presents several challenges.

Scenario 1: measuring on a horizontal surface; with ideal temperature, using a standardized tape, and starting point and end point are less than 100 feet apart. Required tools: steel tape, tension meter.

Scenario 2: Starting point and end point are more than 100 feet apart, at a horizontal surface. Required tools: steel tape, tension meter, chaining Pin (s), range pole (i.e. a third team member is required to secure alignment)

Scenario 3: Temperature is different from 68deg F (standard temperature). Additional math skills are required to correct for the difference in temperature.

Scenario 4: The tape is not standardized and has an error in measurement of $E/100$ ft. Math skills are required to correct for the instrument error.

Scenario 5: Repeat the previous iterations but on non-horizontal surfaces, including steep slopes and rough terrains.

Figure 1 illustrates how this simple surveying exercise has the potential to improve students' representational fluency in math. In frame 1 Prof. Acute (the math tutor) explains the law of cosines using a symbolic representation as well as a verbal explanation; in frame 2 he illustrates the law of cosines and Pythagorean theorem using numeric, graphic and spoken word representations. Frames 3-5 are 3D visualizations (concrete representations) that illustrate the application of the law of cosines and Pythagorean theorem in the context of surveying, and highlight the

difference between horizontal distance and line of sight distance; frame 6 shows one of the surveying exercises (real world application) presented to the students.

Multiple interconnected representations are also used to explain more complex math/trigonometry /geometry concepts such as the one included in educational module 4 (Triangulations and Coordinate Calculations). In module 4 students are required to calculate the coordinates of a point of interest C from two points of known coordinates A and B (Figure 2).

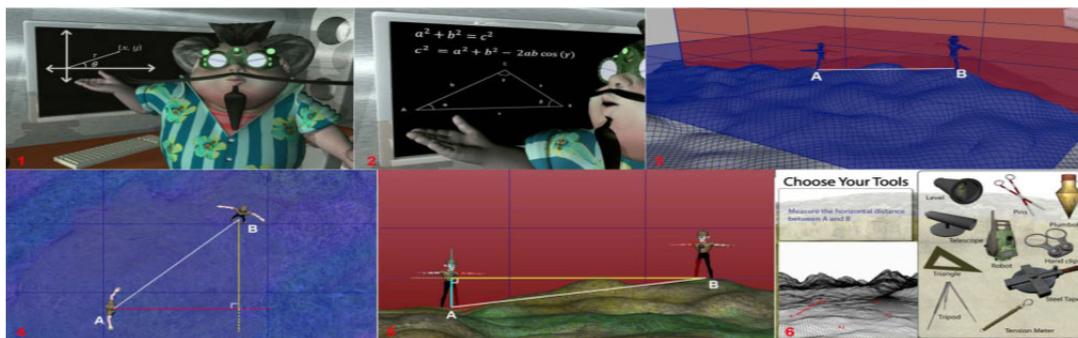


Figure 1. Six screenshots of the 3D VELS showing multiple representations of the law of cosines and Pythagorean Theorem in the context of surveying. A higher resolution version of this image is available at: (omitted for blind review purpose)

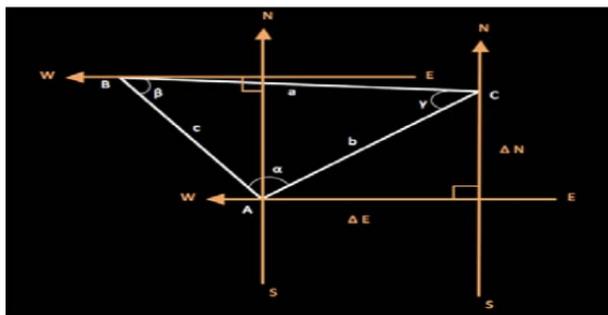


Figure 2. Frame 1: Symbolic and graphical representations of the math concepts used to calculate the coordinates of a point of interest C from two points of known coordinates A and B

In order to calculate the coordinates of point of interest C, the students need to know the distance between point C and the two other points (A and B), as well as the horizontal angle between the three points. Figure 2 includes the symbolic and graphical representations of the required math concepts drawn on the blackboard by Prof. Acute. The image shows (1) the law of cosines which is applied to calculate the angles of the triangle and (2) the trigonometric formula of inverse tangent which is used to calculate the angle [North A,B] using the known coordinates of the two points A and B. Prof. Acute highlights the right triangle using the axes of reference with North-South and East-West passing by the two points A and B. Another set of axes of references North-South and East-West passing by the two points A and C is drawn to highlight another right triangle that will be used to calculate the coordinates of points C. In Figure 3, frames 1-3 show 3D visualizations of the same math concepts in the virtual environment; Frames 4-7 show a concrete application of the math concepts in the context of surveying , e.g. performing an accurate measurement in the field.

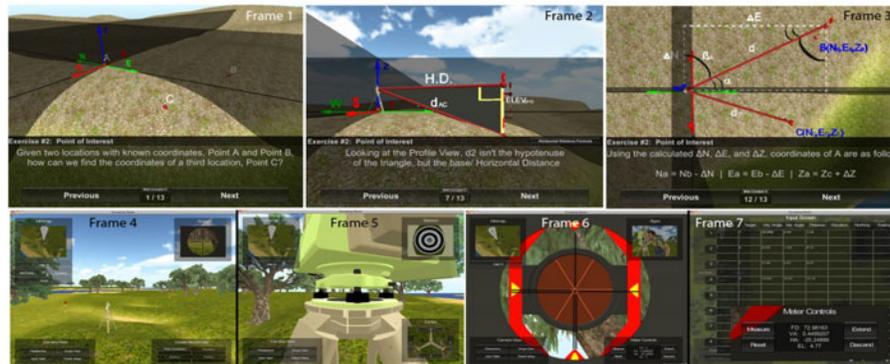


Figure 3. Frames 1-3 are 3D visualizations of the math concepts in the virtual environment; frames 4-7 are screenshots from the interactive VELs. Frames 4-7 show a concrete application of the math concepts in the context of surveying , e.g. performing an accurate measurement in the field. Specifically, frame 4 shows the open field where the surveying exercise is performed (the student selects the location where to set up the instrument and the points of interest); frame 5 shows a close up on the machine; frame 6 shows a close up on the target where the measurement is taken; frame 7 shows the input table where the student records the measurements.

DISCUSSION AND FUTURE WORK

To date we have conducted formative and summative studies of all VELs modules to determine whether they increase students' declarative and procedural knowledge of surveying concepts and practices (Dib et al. 2014; 2011a; 2011b). Currently, we are conducting formative and summative evaluations to determine whether the VELs improves representational fluency in mathematics and support students' transfer of these mathematical concepts in the context of learning surveying. Formative studies include: one-to-one evaluation with subject matter experts (e.g. experts in mathematics and representational fluency in mathematics and surveying educators), small group evaluation with learners, and field trial and observation with groups of learners. Summative studies include pre-post test with a control group. To assess the development of representational fluency, we have developed a criterion-referenced construct-valid instrument using a modeling approach that combines classes and continua of thinking. By ensuring that we have thorough system design requirements as well as meaningful student performance data, we should be able to investigate the relationship between system features and student learning and transfer. These studies are being conducted at Purdue University; the subjects are undergraduate students enrolled in the Building Construction Management (BCM) program in the College of Technology, faculty from the Department of Mathematics, and other BCM faculty who teach surveying. Findings of the studies will be reported in a future publication.

Although some authors have documented that Virtual Reality experiences provide advantages over more traditional instructional methods (Ainge 1996 and Song 2000), studies of Virtual Reality projects are early and still relatively few and a

need exists for investigations of Virtual Reality in the undergraduate classroom (Strangman 2003). Therefore, we believe that the project described in the paper, focused on developing and testing an interactive virtual environment, including virtual manipulatives, for developing students' representational fluency as a part of learning surveying, has the potential to advance our understanding of how to design and use virtual manipulatives and learning environments that a) substantively incorporate 3D virtual reality technology, b) are immersive, meaning that educational interactions occur within the environment making the environment a "place", c) are designed information spaces in which the information is explicitly represented and, the educational interactions occur so that, and students are not only active, but actors, i.e., they co-construct the information space, d) overlap and are an extension of the physical environment, thereby possessing the potential to enrich classroom activities, and e) can integrate heterogeneous technologies and multiple pedagogical approaches.

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REFERENCES

- Elgin, R., (2007). Surveyors Report: The Demise of Basic Surveying Mathematics. *The American Surveyor* - Wed May 30, 2007.
- Kaput, J. J. (1989). Linking representations in the symbol systems of algebra. In S. Wagner and C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 167–194). Reston, VA: National Council of Teachers of Mathematics.
- Knuth, E. (2000). Student understanding of the Cartesian connection: An exploratory study. *Journal for Research in Mathematics Education*, 31, 500–508.
- Dark, M. J. (2003). A models and modeling perspective on skills for the high performance workplace. In R. A. Lesh and H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 279–293). Mahwah, NJ: Erlbaum.
- English, L. (2007). Mathematical modelling with young learners. In S. J. Lamon, W. A. Parker, and S. K. Houston (Eds.), *Mathematical modelling: A way of life* (pp. 3–18). Chichester, England: Horwood.
- Dillenburg, P. 2000. Virtual Learning Environments. *Proc. of EUN Conference 2000 - Workshop on Virtual Learning Environments*.
- Hmelo, C., and Williams, S. M. (Eds.). (1998). Special issue: Learning through problem solving. *The Journal of the Learning Sciences* 7, 3(4).
- Bransford, J. D., Brown, A. L. and Cocking, R. R. 1999. *How People Learn: Brain, Mind, Experience, and School*. Washington DC: National Academies Press. Available at <http://books.nap.edu/catalog/6160.html>

- Kangassalo, M. (1994). Children's independent exploration of a natural phenomenon by using a pictorial computer-based simulation. *Journal of Computing in Childhood Education*, 5(3/4), 285-297.
- Zietsman, A.I., and Hewson, P.W. (1986). Effect of instruction using microcomputer simulations and conceptual change strategies on science learning. *Journal of Research in Science Teaching*. 23. pp. 27-39.
- Gorsky, P., and Finegold, M. (1992). Using computer simulations to restructure students' conceptions of force. *Journal of Computers in Mathematics and Science Teaching*, 11, 163-178.
- Mannix, M., 2000. The virtues of virtual labs. *ASEE Prism* – Sep. 2000. Available at: <http://www.prism-magazine.org/sept00/html/toolbox.cfm>
- Karweit, M. (2010). A Virtual Engineering/Science Laboratory Course Available at: <http://www.jhu.edu/~virtlab/virtlab.html>
- Smart Structures Technology Laboratory (SSTE) at UIUC (2008). Virtual Laboratory for Earthquake Engineering. Available at: <http://sstl.cee.illinois.edu/>
- Kuo, H.L., Kang, S.C., Lu, C.C., Hsieh, S.H., Lin, Y.H., (2007). Feasibility study: Using a Virtual Surveying Instrument in Surveying Training. ICEE 2007, Coimbra Portugal.
- Ellis, R.C.T., Dickinson, I., Green, M. and Smith, M. (2006). The Implementation and Evaluation of an Undergraduate Virtual Reality Surveying Application. BEECON 2006 Built Environment Education Conference, London, 2006.
- Mills, H., Barber, D., (2008). A virtual surveying field-course for traversing. E-learning in Surveying, Geo-information Sciences and Land Administration. FIG International Workshop 2008, Enschede, The Netherlands, 2008.
- Dib, H., Adamo-Villani, N. and Garver, S. (2014) (accepted-in press). An interactive virtual environment for learning differential leveling: development and initial findings. *Advances in Engineering Education (ASEE)*.
- Dib, H. and Adamo-Villani, N. (2011a). An e-tool for undergraduate surveying education: design and evaluation. *ICST Transactions on e-Education and e-Learning*, July–September 2011 | Volume 11 | Issues 7–9 | e5.
- Dib, H. and Adamo-Villani, N. (2011b). An innovative software application for Surveying Education. *Journal of Computer Applications in Engineering Education (Wiley)*. October 2011, <http://onlinelibrary.wiley.com/doi/10.1002/cae.20580/full>
- Ainge, D. J. (1996). Upper primary students constructing and exploring three dimensional shapes: A comparison of virtual reality with card nets. *Journal of Educational Computing Research*, 14(4), 345-369.
- Song, K., Han, B., and Yul Lee, W. 2000. A virtual reality application for middle school geometry class. Paper presented at the International Conference on Computers in Education/International Conference on Computer-Assisted Instruction, Taipei, Taiwan.
- Strangman, N., and Hall, T. (2003). Virtual reality/simulations. Wakefield, MA: National Center on Accessing the General Curriculum. Retrieved March 2007 from http://www.cast.org/publications/ncac/ncac_vr.html