Optimisation of Grillage-Type Foundations

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Summary

The mathematical models for optimisation of grillage-type foundations are presented. Minimising of maximum in absolute value vertical reactive force, bending moment, and reaction-bending moment together is sought employing methods of finite elements, analytical sensitivity analysis, and mathematical programming. Present models and computer code are implemented in the software MatrixFrame®. Solutions of a number of problems demonstrate the validity of proposed algorithms.

1. Introduction

All parts of buildings should be designed and built optimally and thrifty as much as the conditions of safety and comfort allow. In the design of grillage-type foundations this simply means that, firstly, the cross-section of grillage is uniform in all the structure, and secondly, piles supporting the grillage are uniform over all structure, but are placed plausible, not at equal distances from each other. In order to optimally utilise the steel framework of grillage, the bending moments should be uniformly distributed over the structure or, at worst, maximum positive moments should match the minimum ones. For the support reactions in piles, obviously, all reactions should be as small as possible and uniform.

Thus, the design of economical grillage foundations inevitably is related with optimisation of initial scheme.

The paper deals with the aforementioned problems. We tried to pose the optimisation problems, to define the solution methods, etc, up to the introduction into commercial codes.

2. Statement of problem

The optimisation problem is stated as follows:

Minimise (over feasible shapes) maximum P (over structure and load cases)

with P being the parameter to be optimised.

Two optimisation problems are to be examined: when parameter is maximum bending moment at some points of structure, and maximum vertical reactive force at supports. The feasible shape of structure is defined by the type of certain supports (unmoveable support, spring-support, or support with a given displacement), the given number of different cross-sections and different materials in the structure. During optimisation process new moveable supports may appear in the structure, the old supports may merge, however the type of existing supports has to be retained.

The problems should be solved in statics and in linear stage.

Clearly, both problems are highly non-linear. Our choice is to use robust and reliable methods: finite element method for static analysis and linear mathematical programming for optimisation. Thus, the problems have to be solved iteratively and are converted to a sequence of approximately linear problems of an optimal re-design. In each iteration the current shape is changed to a better neighbouring shape. The solution requires three steps:

- finite element analysis
- sensitivity analysis with respect to the co-ordinates of supports
- optimal re-design with linear programming .

Further, the minimum-maximum problem is converted to a pure minimum problem with constraints by treating P_{max} as unknown subject to constraints that P_{max} limits the magnitudes of parameter P everywhere in the structure and for all load cases when design changes Δt_i are performed:

$$P(x) + \sum_i P(x),_{t_i} \Delta t_i - P_{max} \le 0 \tag{1}$$

for the total structural space x. The comma here and below means the differentiation.

For computational reasons a length constraint $L = \bar{L}$ is also included:

$$L + \sum_i L,_{t_i} \Delta t_i - \bar{L} = 0 \tag{2}$$

Several possibilities exist in the choice of design parameters t_i on which the structure shape depends. Our choice is to use the most evident from the engineering point of view design parameters: nodal co-ordinates of all (or a chosen set of) supports.

3. Optimisation technique

With reference to [1, 2] let us shortly describe the optimisation procedures.

Two absolute limits sets (maximum, nonnegative and minimum, nonpositive) on all design co-ordinates status $\mathbf{T}$: $\mathbf{T}^{\mathbf{max}}$ and $\mathbf{T}^{\mathbf{min}}$ are led up according to existing design restrictions or other considerations. In any case the design variable cannot exceed these limits. For the first solution step, current design variables status $\mathbf{T} = \mathbf{0}$. The absolute limits may differ from one design variable to other, however the maximum absolute move limits must be positive, and the minimum ones negative. Further, the move limits on the design variables alterations $\Delta\mathbf{T}$ per one iteration are led up, again maximum and minimum. These move limits may vary from one design variable to another and have to be adjusted to the extent of non-linearity of problem so that Simplex' predictions on the future behaviour of the structure do not differ remarkably from finite element solution. In general, move limits should be gradually shrunk as the design approaches the optimum. The accuracy of the approximation is required to be higher when we get close to the optimum because the gains are small and can be swamped by approximation errors. After introduction of intermediate always positive variables $\Delta\mathbf{T}^{+}$ and $\Delta\mathbf{T}^{-}$ such that :

$$\Delta\mathbf{T} = \Delta\mathbf{T}^{+} + \Delta\mathbf{T}^{\mathbf{min}} \, . \tag{3}$$

$$\Delta\mathbf{T}^{+} \leq \Delta\mathbf{T}^{\mathbf{max}} - \Delta\mathbf{T}^{\mathbf{min}} \, . \tag{4}$$

$$\Delta\mathbf{T}^{+} + \Delta\tilde{\mathbf{T}} = \Delta\mathbf{T}^{\mathbf{max}} - \Delta\mathbf{T}^{\mathbf{min}} \, . \tag{5}$$

all necessary conditions to the Simplex procedure are satisfied.

Finally, the problem formulation for mathematical programming is:

Minimise P_{max}

with constraints:

level of P everywhere in the structure $\leq P_{max}$,

design changes do not exceed move limits, and design status does not exceed absolute limit ;

length of model is constant .

Considering only the first derivatives in Taylor's expansion, the first constraints at the nodal points of structure become

$$\mathbf{P} + [P]_{,\mathbf{T}} \, \Delta\mathbf{T} - \mathbf{P}_{\mathbf{max}} \leq \mathbf{0} \, , \tag{6}$$

or avoiding the inequality

$$[I] \, \tilde{\mathbf{P}} - \mathbf{1} \, P_{max} + [P]_{,\mathbf{T}} \, \Delta\mathbf{T}^{+} = - \mathbf{P} - [P]_{,\mathbf{T}} \, \Delta\mathbf{T}^{\mathbf{min}} \, . \tag{7}$$

The second group of constraints in matrix notation for all design variables is:

$$\Delta\mathbf{T}^{+} + \Delta\tilde{\mathbf{T}} = \Delta\mathbf{T}^{\mathbf{max}} - \Delta\mathbf{T}^{\mathbf{min}} \tag{8}$$

while the third one is as follows:

$$L + \sum [L^{e}]_{,\mathbf{T}} \Delta\mathbf{T} = \bar{L} \tag{9}$$

where the sum covers only the active elements, i.e. including the current design variable as a node of element. In the first iteration $L = \bar{L}$.

4. Finite element. Matrices for sensitivity analysis

Simple two-node beam element with 4 d.o.f.'s [3] has been implemented in analysis. Nodal d.o.f.'s of element are:

$$\mathbf{u} = \{ w_i , \theta_i , w_j , \theta_j \}^T \tag{10}$$

w_k and Θ_k, $k = i,j$ being deflection and rotation, positive counter-clockwise, accordingly.

The interpolation functions can be found in [3].

Bending moments at nodes, positive when cause the "positive" layers of a finite element experience tension, compile the element stress vector. Flexural rigidity and interpolation functions relates moment to the deflection:

$$M = -E I w_{,xx} = E I \sum_i N_{i,xx} u_i \tag{11}$$

After the nodal displacements are obtained, the reactive forces are available according to:

$$R_i = \sum_j \mathbf{K}_{ij} \mathbf{u}_j \tag{12}$$

K being the stiffness matrix.

Finite element can be loaded by nodal forces and moments, positive counter-clockwise, and by concentrated loads, moments, distributed (of triangular shape) loadings inside the element. Distributed loading is modified to the statically equivalent loads and moments acting at the end-points of loading. Later on these components as well as all other internal concentrated loads and moments are translated to the nodes of finite element according to general relations of finite element method yielding nodal loads vector **P**.

As seen from (1), the sensitivity (i.e., derivatives with respect to nodal co-ordinates) of bending moments and reactive forces is the must for optimisation.:

$$\mathbf{M}_{,x_k} = -E I \sum_i \left(N_{i,x x x_k} u_i + N_{i,x x} u_{i,x_k} \right), \tag{13}$$

$$\mathbf{R}_{,x_k} = [K]^a_{,x_k} \mathbf{u}^a + [K]^a \mathbf{u}^a_{,x_k} \tag{14}$$

with superscript a standing for ensemble.

The derivatives of nodal displacements are obtained by solution of general sensitivity analysis:

$$[K]^a \mathbf{u}^a_{,x_k} = \mathbf{P}^a_{,x_k} - [K]^a_{,x_k} \mathbf{u}^a \tag{15}$$

The procedure for derivative of element stiffness matrix from which matrix of ensemble is $[K]^a{,}x_i$, composed, is as follows: replace L with xj-xi, detect whether k is i^{th} or j^{th} node of an element, and obtain $[K]{,}x_i$ or $[K]{,}x_j$, respectively. Thus, only the element possessing node k renders non-zero stiffness derivatives.

Similar procedures are valid for derivatives of forces and reactions. Due to relatively simple interpolation functions all sensitivity analysis is performed analytically with computer algebra.

5. Program

The finite element computational procedure, sensitivity analysis and optimal re-design via linear programming form the programs kernel which is supplemented with pre- and post-processing capabilities. Kernel is written in Fortran 90 while pre- and postprocessors in C++.

The main features of kernel are:

- the program is "one button click program"
- input is: lengths of beams composing grillage, data on joints of beams, data on nonmovable supports, if those are led in the schema due to some considerations of program user; characteristics of materials and cross-sections, loadings, allowables: support reaction, deflection in beams, distance between adjacent supports
- output is: positions of supports, reactions in supports, bending moments in beams

The initial finite element mesh is prepared automatically, leading up nodes at support places, jumps of material and cross-sections properties, etc. The main problem inherent to the use of linear mathematical programming is that in the case of numerous supports Simplex inevitably leads to the local minimum point. Our solution to this is to use quasi-optimal initial schema for optimisation procedure. This part of kernel is most complex program which analyses loadings, geometry, materials and renders trial supports placement schema for optimization. Later on finite element mesh is generated, again automatically, pointing "master nodes" which are allowed to move over the schema freely and adjusting their move limits to the level of problems nonlinearity. Number of supports can be increased in optimisation process, if allowable reactions were not achieved.

Separate beams of grillage or separate parts of grillage may be optimised independently if program user wishes.

Key in the optimisation of grillage is optimisation of a single beam. For the whole grillage the "upper beams" which reside on the "lower beams" are distinguished. The beams-joints are simulated simply as supports (for upper beams) or concentrated forces (for lower beams). Sensitivity of optimised parameter in upper beam depends on the optimisation results of lower beams, and vice versa, therefore grillage optimisation procedure is included into additional iteration loop to achieve required accuracy.

Program has capabilities to minimise support reactions, bending moments in beams, or reactions/bending moments together. The satisfactory merit function for the last problem was not found; an engineering approach has been employed instead: program begins and proceeds with optimisation of reactions until allowable reaction is achieved, then shifts to the minimisation of moments. Provided allowable reaction is exceeded, backward shift occurs, etc.

6. Numerical examples

A number of numerical examples demonstrate the capabilities of proposed model. For the sake of transparency optimisation examples of a single grillage beam are presented here. Two examples deal with optimisation of vertical support reactions in foundations beam. The loadings in all examples are chosen so that the obtained results would be simply comparable with in advance known optimal shapes of beams.

All data are given in the figures 1 – 2.

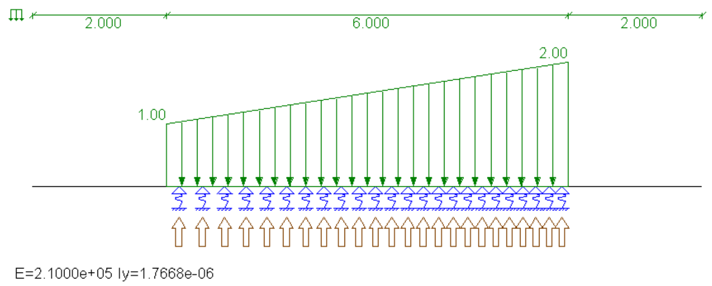

Fig. 1. 1D beam with trapezoidal load

Example Nr.1 :

1D beam loaded with trapezoidal load. Allowable support reaction - 0.4. Spring value for support – 8.0. Optimisation method - supports reaction optimisation. In case of 23 spring supports supports reactions values of Rmax = 0.399, Rmin = 0.378 are reached.

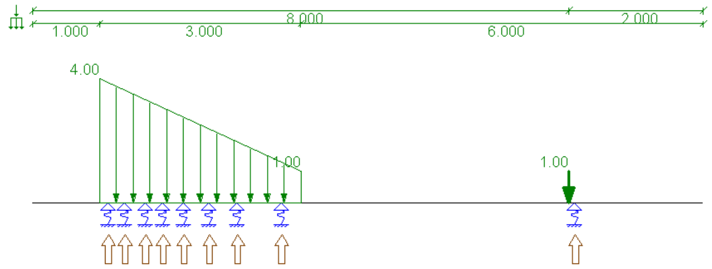

Fig. 2. 1D beam with trapezoidal and concentrated loads

Example Nr.2 :

1D beam loaded with trapezoidal and concentrated loads. Allowable support reaction - 1. Spring value for support – 8.0. Optimisation method - supports reaction optimisation. In case of 9 spring supports supports reactions values of Rmax = 0.974, Rmin = 0.925 are reached.

7. Conclusions

Proposed methods deliver acceptable results. The main problem is, linear mathematical programming in case of numerous design variables inevitably leads to the local minimum. Solution to this is to start from near-optimal initial schema which is generated by special programs analysing the loading conditions and given topology of grillage. Another alternatives are the nonlinear programming, and global optimization methods. Research in these fields is in process.

8. References

[1] [1] Pedersen P., "Design for minimum stress concentration – some practical aspects", *Structural Optimisation*, Kluwer Academic, 1988, pp. 225-232.

[2] [2] Belevičius R. "Shape optimisation of laminated orthotropic plate structures", *Mech. of Composite Mater.*, No. 29, 1993, pp. 537-546.

[3] [3] Cook R. D., Malkus D. S., and Plesha M. E., *Concepts and applications of finite element method*, John Wiley & Sons, 1989, 630 pp.