

# An Strategic Analysis of Urban Growth Controls

Dolores Garcia

Departament D'Economia i Empresa  
Universitat de les Illes Balears  
07071 Palma de Mallorca, Spain

Phone: +34 971 173242

Fax +34 971 173426

e-mail: dolores.garcia@uib.es

Preliminary version

“Paper presented at the 8th European Real Estate  
Society Conference –ERES Alicante, 2001”,  
in Alicante, 27–29 June 2001

May 2001

## **Abstract**

The welfare economics of urban growth controls and other land use regulations have received an increasing deal of attention in recent years, especially at the theoretical level. This paper analyzes two types of growth controls in the context of a closed system of interdependent cities where utility is determined endogenously. Thus, it concentrates on how the use of population growth controls and, alternatively, the use of taxes on housing consumption, affect utility levels, taxes revenues and city size, in a simple context in which households' utility is not affected by environmental amenities. Several scenarios are analyzed, with particular attention to the emerging equilibria when strategic interaction between cities takes place, both considering static and dynamic horizons. It is shown that cooperation between jurisdictions and the subsequent choice of stringer population controls and higher taxes constitute the equilibrium solution when interaction is to occur along infinite periods.

# 1 Introduction

The use of urban planning instruments is common to most western countries. Among European countries planning systems vary a great deal, but the presence of the public sector is habitual along the several stages of the planning process. In Spain there has been a long tradition of intervening the land market and the planning process, and instruments such as density levels and the delimitation of land suitable for development are jointly used. The role of land-use controls as a means to guide urban development and restrict urban growth has been long and widely analyzed in the urban literature. From the viewpoint of resident households, the economic justification for the introduction of growth control relies mostly on the alleged relationship between the urban size and the existence of external costs, related for instance to the appearance of congestion or to the loss of outer landscapes. In this sense, restricting the urban size may lead to increases in welfare. This is the approach followed by the so-called *amenity-creation models*, by which planning restrictions would improve urban amenities, what ultimately translates into increases in land rents [Brueckner (1990); Engle, Navarro and Carson (1992)].

However, in the theoretical urban economics literature it seems to predominate the idea that actual planning restrictions are welfare-worsening, even though they may correct externalities [Fischel (1990); Anas, Arnott and Small (1998)]. Even when they result successful in preserving the urban environment, they are supposed to achieve the end at too high costs compared to alternative instruments such as taxes or impact fees, that truly distort residents' decisions [Brueckner (1997); Brueckner (2001)]. Recently, a new line of research has regarded urban planning decisions as the result of the strategic interaction among cities. This approach allows for the emergence of restricted city sizes even though urban growth involve external costs [Helsley and Strange (1995), Brueckner (1998)].

This paper analyzes the welfare effects of planning restrictions, under different scenarios. It uses the bid-rent framework to try to analyze two types of growth controls, namely population regulations and a tax that burdens housing consumption. It is assumed that the utility function of residents is not affected by any urban characteristic such as density or the city size. Thus, utility only depends upon the consumption of land and all other private non-land goods. The model consists in a closed system of three interdependent cities where utility is determined endogenously. Two types of households can

be differentiated attending to their income levels, and both types migrate freely and at zero cost from one city to another. The effects when one or two of the cities impose some type of regulation are analyzed, and quantity and price instruments are considered, in the form of population controls and a tax on housing consumption. In particular, it will be considered that cities may impose *endogenous* land use regulations, in the sense that they maximize certain objective function for the local planner. We will consider that local communities maximize the fiscal revenue arising from either from population controls or taxes.

Special attention is paid to the scenarios in which two of the cities may impose some form of controls, that will be strategically chosen. The equilibrium strategies will be obtained for the cases in which cities use population controls and taxes, something that has already done in the literature. In this respect, the distinct feature in this paper is to extend the results to a dynamic context and to allow for the possibility of cooperation between jurisdictions.

The subsequent sections show the following contents. Section 2 describes the main features of the model, and shows the equilibrium conditions without planning restrictions. In section 3, the effects of endogenous population controls are analyzed, differentiating two scenarios: the first, when a single city imposes the control, and the second, when they take into account the decisions of rival communities and decide strategically. Thus, we investigate about the equilibrium strategies both for a one-period context and a multi-period scenario. Section 4 undertakes the effects of price controls in the form of taxes on housing. It covers again the case when decisions are taken separately or considering other cities' choices, in static and dynamic frameworks, too. Section ?? shows the differences in tax collection outcomes that results from using population controls or taxes. The final section summarizes the main outcomes and conclusions of the analysis.

## 2 The basic model

The benchmark model here has 3 cities, denoted by superscript  $i$ ,  $i = A, B, C$ . City  $A$  and  $B$  may impose growth restrictions, while in all cases city  $C$  simply accommodates all coming residents. Cities are supposed to be linear and with a width of 1. All residents work at the Central Business District (CBD), located at an extreme of the city. Individuals must commute to the

city centre at a cost  $T(r)$ , where  $r$  represents every possible distance from home to the CBD. Transportation costs increase linearly with distance in all cities, so  $T^i(r) = tr$ , and transportation costs do not vary across cities. Households rent a fixed amount of housing and respond to utility differentials by migrating from one city to another at zero cost.

In the simplest model, households' utility depends upon the consumption of housing space  $s$  and a composite good  $z$ , which is also the numéraire. Utility is then

$$u^i = u(s_j, z_j) \quad (2.1)$$

where  $u_s > 0$  and  $u_z > 0$ . At every city residents must pay housing rents to absentee landowners, and the rental price of housing per period of time is denoted by  $R^i(r)$ . The housing market is assumed to be competitive.

All households have identical tastes, but may differ in income level, denoted by the subscript  $j$ ,  $j = 0, 1$ . There are two levels of wealth among residents. There is a population of  $N_0$  with an income level of  $Y_0$ , and  $N_1$  residents who receive  $Y_1$ , with  $Y_0 < Y_1$ . Income levels are supposed to be exogenous and the role of firms in the city is not considered. Individuals spend their income between the composite good  $z$ , housing space  $s$  and transportation. Housing consumption is fixed, with richer people consuming a normalized quantity  $s_1 = 1$  and poorer ones consuming  $s_0 = \alpha$ , with  $\alpha < 1$ . Since housing space is determined exogenously, the only variable that affects the utility level achieved by households will be the consumption of all other private goods different from housing, that is  $z$ . The respective budget constraints of both types of households can then be expressed as:

$$Y_1 = z_1(1, u_1) + R_1^i(r) + tr \quad (2.2)$$

$$Y_0 = z_0(\alpha, u_0) + \alpha R_0^i(r) + tr, \quad (2.2')$$

or, in terms of the housing bid-rents:

$$R_1^i(r) = Y_1 - tr - z_1 \quad (2.3)$$

$$R_0^i(r) = \frac{Y_0 - tr - z_0}{\alpha}. \quad (2.3')$$

Since transportation costs increase proportionally with distance, the housing rent or housing *bid-rent* decreases linearly with distance to the CBD. For each income level, there exists a family of housing bid-rent functions that

correspond to different utility levels. For individuals to be in equilibrium and indifferent among locations within the city, housing rents must vary as described by the housing-bid rent function above. Thus, at a larger distance from the CBD, higher transportation costs are compensated by a smaller housing rent, so that all individuals belonging to the same income group can attain identical utility levels independently of the particular location.  $R_0^i$  is steeper than  $R_1^i$ , this implying that the less wealthy locate closer to the CBD. The inner segment where poorer households locate has a radius of  $\widehat{r}^i$ . Wealthier residents locate in the outer segment comprised between radius  $\widehat{r}^i$  and  $\bar{r}^i$ , where  $\bar{r}^i$  represents the edge of the city.

Housing is produced from land and capital, according to the production function  $H(l, k) = lk$ , which shows constant returns to scale. Combining  $k$  units of capital and  $l$  units of land yields  $lk$  units of housing <sup>1</sup>. The rental price of capital is denoted by  $P$ . It will be assumed that both types of housing require the same amount of capital investment.  $L_j^i(r)$  represents the rental price of land, also variable with distance. The relationship between housing rent and land rent is given by:

$$R_j^i(r) = \frac{L_j^i(r)}{k} + P, \quad (2.4)$$

or

$$L_j^i(r) = k[R_j^i(r) - P] = k\left[\frac{Y_j - tr - z_j}{s_j} - P\right]. \quad (2.5)$$

Finally, at radius  $\widehat{r}^i$  in all cities land rents must coincide, that is

$$k\left[\frac{Y_0^i - t\widehat{r}_i - z_0}{\alpha} - P\right] = k[Y_1 - t\widehat{r}_i - z_1 - P]. \quad (2.6)$$

At all locations, land is allocated to that activity yielding the highest return.

## 2.1 Equilibrium without planning restrictions

Equilibrium in the land market involves several conditions. Firstly, total population  $N_0$  and  $N_1$  must be accommodated within the boundaries of the

---

<sup>1</sup>Accordingly, variable  $k$  denotes density, since it refers to the number of housing units per unit of land.

cities. Considering that cities are linear and that housing space has been fixed, this implies that

$$\widehat{r^A} + \widehat{r^B} + \widehat{r^C} = \frac{\alpha N_0}{k} \quad (2.7)$$

and

$$\overline{r^A} + \overline{r^B} + \overline{r^C} = \frac{\alpha N_0 + N_1}{k}. \quad (2.8)$$

Secondly, if residents are perfectly mobile, utilities in all cities must equal for each type of household. Since housing consumption is fixed and identical for individuals in the same income range, for them to be indifferent between cities their consumption of non-housing goods must also be the same, that is  $z_j^i = z_j$ . Finally, in a context without planning restrictions it is required that in all cities the urban land rent equals the value of the best alternative use at the city limit, usually considered to be the agricultural. For simplicity the agricultural value is supposed to be zero. Then  $L_1(\overline{r^i}) = 0$ , or

$$Y_1 - t\overline{r^i} - z_1 - P = 0. \quad (2.9)$$

From equation 2.9 it can be derived that  $\overline{r^A} = \overline{r^B} = \overline{r^C} = \overline{r}$ , and from 2.8, it results:

$$\overline{r} = \frac{\alpha N_0 + N_1}{3k}. \quad (2.10)$$

Thus, in the non-restricted equilibrium population equally distributes among cities, and the less wealthy occupy an identical inner radius of

$$\hat{r} = \frac{\alpha N_0}{3k}. \quad (2.11)$$

Combining 2.9 and 2.10, the amount of  $z$  consumed by individuals with income  $Y_1$  is:

$$z_1 = Y_1 - P - \frac{t}{3k}[\alpha N_0 + N_1]. \quad (2.12)$$

To obtain  $z_0$ , conditions 2.6, 2.11 and 2.12 are used, resulting in

$$z_0 = Y_0 - \alpha P - \frac{\alpha t}{3k}[N_0 + N_1]. \quad (2.13)$$

Notice that in the absence of negative environmental effects of crowding, households are better off the highest density levels, since that allows transportation costs savings and does not provoke external costs.

The equilibrium utility levels would result:

$$u_0^m = u(\alpha, Y_0 - \alpha P - \frac{\alpha t}{3k}[N_0 + N_1]) \quad (2.14)$$

and

$$u_1^m = u(1, Y_1 - P - \frac{t}{3k}[\alpha N_0 + N_1]). \quad (2.15)$$

Since the following sections all ignore externalities, we can simply concentrate in the consumption levels of  $z$  to analyze how planning controls affect utility levels.

### 3 The effects of population controls

In this section the planning instruments are population controls that restrict city size. The choice of the appropriate city size is endogenous in the sense that it maximizes aggregate urban land rents, an objective function commonly considered in the urban literature. Two scenarios are considered. First, the case where only one city in the system restricts its size; secondly, it is considered that two of the cities impose population controls and they decide strategically.

#### 3.1 Equilibrium with one controlling city

Assume now that city  $A$  imposes an urban population control that restricts city size, and that all excluded households can be accommodated in cities  $B$  and  $C$ . There, the condition that urban land rent equals zero at the city limit continues to be valid. Similarly, land rents must equal at  $\widehat{r}^i$ , so 2.6 at page 4 still applies in the restricted case. Now, using 2.8 it results

$$z_1 = Y_1 - P - \frac{t}{2}[\frac{\alpha N_0 + N_1}{k} - \overline{r^A}], \quad (3.1)$$

where  $\overline{r^A}$  is now a choice variable for the local government in city  $A$ .



From 2.6 in page 4 for all cities, and since  $z_j$  will be the same in equilibrium, it must be the case that  $\widehat{r}^i$  coincide in all cities, and then the introduction of the population control does not alter the size of the inner segment where poorer households live. Then,

$$\widehat{r}^i = \frac{\alpha N_0}{3k}. \quad (3.2)$$

This result suggests that when city  $A$  imposes an urban population control, in the resulting equilibrium the poorer residents continue to split between the controlled and uncontrolled cities, and the size of the inner segments does not vary. Although it has been assumed that both types of households are mobile, the previous result implies that, in practice, some residents do not relocate when growth controls are imposed in one of the cities and that only wealthier households finally migrate. But this is the logical result when considering the assumption that  $z_0$  must be the same independently of the city. Since the relative steepness of land-rents functions does not change in the regulated situation, then  $\widehat{r}^i$  does not modify with the introduction of the population control.

Simplifying and using 2.7 it can be obtained that

$$z_0 = Y_0 - \alpha P - \frac{\alpha t}{2k} \left[ \frac{(\alpha + 2)}{3} N_0 + N_1 - k\overline{r^A} \right]. \quad (3.3)$$

With the use of a population growth control in city  $A$ ,  $z_0$  and  $z_1$  are negatively affected. The positive signs of the partial derivatives of  $z_j$  with respect to  $\overline{r^A}$  show that the consumption of all other goods increases with  $\overline{r^A}$ , that is, the less restrictive the control is. Since housing consumption is exogenously determined, the population control makes residents worse off in this simple context without environmental externalities.

The above results apply whatever the values of  $\overline{r^A}$ . Consider now the particular case when the population control introduced is endogenous, in the sense that it maximizes a particular objective function chosen by the local government. Suppose an objective function consisting in the sum of all land rents in city  $A$ ,  $TR^A$ , land rents that go to absentee landowners. The decision for the local planner consists then in choosing the value of  $\overline{r^A}$  that maximizes

$$\max_{\overline{r^A}} TR^A = \int_0^{\widehat{r^A}} k \left[ \frac{Y_0 - tr - z_0}{\alpha} - P \right] dr + \int_{\widehat{r^A}}^{\overline{r^A}} k [Y_1 - tr - z_1 - P] dr. \quad (3.4)$$

Using the Leibniz' rule to obtain the first order condition for the maximization of aggregate land rents and solving for  $\overline{r^A}$  yields an endogenous growth control smaller than the market equilibrium city size,

$$\overline{r^{A*}} = \frac{1}{4k}[\alpha N_0 + N_1]. \quad (3.5)$$

Finally,  $z_1$  and  $z_0$  can be expressed in terms of the parameters:

$$z_1 = Y_1 - P - \frac{3t}{8k}[\alpha N_0 + N_1], \quad (3.6)$$

and

$$z_0 = Y_0 - \alpha P - \frac{\alpha t}{8k} \left[ \frac{(\alpha + 8)}{3} N_0 + 3N_1 \right]. \quad (3.7)$$

It can be shown that, as expected, introducing the endogenous population control makes both types of residents consume smaller amounts of  $z_0$  and  $z_1$ , and as a result they attain smaller utility levels.

### 3.2 Equilibrium with two controlling cities

Assume now that all cities in the system impose population controls so as to maximize total land rents, and that they are aware that the remaining cities use them as well. In equilibrium all households must be accommodated, and since housing consumption has been exogenously fixed, it is made the assumption that city  $C$  acts passively and accommodates all coming households.

Now  $A$  and  $B$  want to maximize their respective aggregate land rents,  $TR^A$  and  $TR^B$ , but taking into account the rival choice of city size  $\overline{r^i}$ . This implies to search for the best strategy to follow given the actions of all other local governments in the urban system, that is for each city best response function. The problem can be solved as a typical simultaneous game, where the decision variables for cities are population controls.

The size of the segment where the less wealthy households live does not vary, and  $\hat{r} = \frac{\alpha N_0}{3k}$ . Besides, in city  $C$  equation 2.9 in page 5 holds, and the equilibrium levels of  $z_1$  and  $z_0$  can be calculated from:

$$z_1 = Y_1 - P - t \left[ \frac{\alpha N_0 + N_1}{k} - \overline{r^A} - \overline{r^B} \right] \quad (3.8)$$

$$z_0 = Y_0 - \alpha P - \frac{\alpha t}{k} \left[ \frac{(1 + 2\alpha)}{k} N_0 + N_1 - k(\overline{r^A} + \overline{r^B}) \right] \quad (3.9)$$

From above it is shown that, as in the one controlling city case, less restrict population controls lead to higher consumption of  $z_1$  and  $z_0$ .

The objective of city  $A$  is to maximize aggregate land rents  $TR^A$ . However, city  $A$  has to consider city  $B$  choices of  $r^B$ . This is achieved by combining  $z_1$  and  $z_0$  in 3.8 and 3.9 together with the expression of  $TR^A$  in 3.4. From the maximization of the expression above the best response function for city  $A$  is found:

$$\overline{r^A}(\overline{r^B}) = \frac{\alpha N_0 + N_1}{3k} - \frac{\overline{r^B}}{3}. \quad (3.10)$$

Symmetrically, the expression of the best response function of city  $B$  would be:

$$\overline{r^B}(\overline{r^A}) = \frac{\alpha N_0 + N_1}{3k} - \frac{\overline{r^A}}{3}. \quad (3.11)$$

Notice that the optimal  $\overline{r^A}$  for city  $A$  when maximizing  $TR^A$  diminishes the larger  $\overline{r^B}$ . Thus, if city  $B$  fixes a not too stringent  $\overline{r^B}$ , then city  $A$  benefits from choosing a smaller  $\overline{r^A}$ . This suggests that population controls act as strategic substitutes. The land rent sacrificed by excluding a household decreases if a rival community enacts a less stringent population control, and then communities are willing to introduce more stringent population controls. Seemingly, a more stringent control increases the opportunity cost of losing population, and as a result cities choose larger city sizes.

Solving the system with the best response functions for cities  $A$  and  $B$  yields smaller sizes in city  $A$  and  $B$ , but a larger  $\overline{r^C}$ . The expressions for the Nash equilibrium  $\overline{r^A}$  and  $\overline{r^B}$  coincide with the ones they would be choosing if they were individually imposing the control:

$$\overline{r^{comp}} = \overline{r^A} = \overline{r^B} = \frac{1}{4k} [\alpha N_0 + N_1]. \quad (3.12)$$

However, the equilibrium utilities achieved are smaller. Because more cities in the system impose population controls, this leads to a larger number of residents diverted to city  $C$  and consequently to higher land rents. These equilibrium values of  $z_1$  and  $z_0$  in this simultaneous population control game are:

$$z_1 = Y_1 - P - \frac{t}{2k} [\alpha N_0 + N_1], \quad (3.13)$$

and

$$z_0 = Y_0 - \alpha P - \frac{\alpha t}{2k} \left[ N_1 + \frac{(2 + \alpha)}{3} N_0 \right]. \quad (3.14)$$

Finally, the equilibrium land rents could be found. Substituting 3.12 back into the expression for land rent in city A in 3.4, and considering that both cities A and B use optimal growth controls, it is found the expression for total land rents, denoted by  $TR^{comp}$ :

$$TR^{comp} = \frac{t}{288k} [11\alpha N_0^2 + 54\alpha N_0 N_1 + 27N_1^2 + 16\alpha N_0]. \quad (3.15)$$

### 3.3 A cooperative framework

This subsection introduces the possibility that cooperation between jurisdictions exists. To our knowledge, the literature using game theory as a means to explain urban growth controls has so far used a non-cooperative approach. In this subsection it is aimed to explore in which instances cooperation between jurisdictions is plausible, both in a static and in a dynamic context.

In this scenario without *competition*, the first information needed is the city size that cities would choose to maximize aggregate land rents. With no a priori differences between cities, one should expect a symmetric city size, so that  $\bar{r}^A = \bar{r}^B$  under the cooperation agreement. Let  $\bar{r}^{coop}$  denote each individual city size with cooperation. This could be considered a particular case of the problem solved in the previous subsection where competition existed, but now imposing the symmetry condition in the election of city size<sup>2</sup>. The corresponding expressions of  $z_0$  and  $z_1$  would be

$$z_0 = Y_0 - \alpha P - \frac{\alpha t}{k} \left[ \frac{(1 + 2\alpha)}{k} N_0 + N_1 - 2k\bar{r}^{coop} \right] \quad (3.16)$$

and

$$z_1 = Y_1 - P - t \left[ \frac{\alpha N_0 + N_1}{k} - 2\bar{r}^{coop} \right]. \quad (3.17)$$

---

<sup>2</sup>It could be possible, however, that city sizes are different with cooperation, as long as a different agreement is met that leads to the maximum attainable level of total land rents.

To find the optimal city size, for instance in city A, the following expression must be maximized:

$$\max_{\bar{r}^{coop}} TR^A = \int_0^{\hat{r}^{coop}} k\left(\frac{Y_0 - tr - z_0}{\alpha} - P\right)dr + \int_{\hat{r}^{coop}}^{\bar{r}^{coop}} k(Y_1 - tr - z_1 - P)dr. \quad (3.18)$$

It is found that the maximizing population control when cooperating results in

$$\bar{r}^{coop} = \frac{1}{5k}[\alpha N_0 + N_1]. \quad (3.19)$$

Although cooperation leads to the highest land rent attainable, it may well happen that the equilibrium solution is such that cities compete instead of cooperate. Results vary when considering static or dynamic horizons.

### 3.3.1 Cooperation in a static context

For every city, the available strategies in the static case are “cooperating” or “competing”, equivalent to choosing a city size according to the best-response function in equation 3.10.

In the cooperation scenario, both cities choose a city size as that in equation 3.19, and total land rents in each city are:

$$TR^{coop} = \frac{t}{450k}[20\alpha^2 N_0^2 + 90\alpha N_0 N_1 + 45N_1^2 + 25\alpha N_0^2]. \quad (3.20)$$

It can be shown that, as expected,  $TR^{coop}$  results larger than  $TR^{comp}$ . Although  $TR^{coop} > TR^{comp}$ , cities have the incentive to deviate from the cooperation agreement, since  $\bar{r}^{coop}$  is not their best city size choice when the other one cooperates. When A deviates while B cooperates, then

$$\bar{r}^A = \bar{r}^{dev} = \frac{4}{15k}[\alpha N_0 + N_1], \quad (3.21)$$

and

$$\bar{r}^B = \bar{r}^{coop} = \frac{1}{5k}[\alpha N_0 + N_1]. \quad (3.22)$$

Likewise, if the corresponding values of aggregate land rents are denoted by  $TR^{dev}$  and  $TR^{coop'}$ , then:

$$TR^{dev} = \frac{t}{450k}[23\alpha^2 N_0^2 + 96\alpha N_0 N_1 + 48N_1^2 + 25\alpha N_0^2], \quad (3.23)$$

and

$$TR^{coop'} = \frac{t}{450k} [14\alpha N_0^2 + 78\alpha N_0 N_1 + 39N_1^2 + 25\alpha N_0^2]. \quad (3.24)$$

Logically, land rents are larger for the city that deviates from the cooperative agreement, and they are also larger with respect to the cooperative solution. Thus,

$$TR^{dev} > TR^{coop} > TR^{comp} > TR^{coop'}.$$

Figure 3.1 depicts the normal form for the static game just described.

		City B	
		Cooperate	Compete
City A	Cooperate	$TR^{coop}, TR^{coop}$	$TR^{coop'}, TR^{dev}$
	Compete	$TR^{dev}, TR^{coop'}$	$TR^{comp}, TR^{comp}$

Figure 3.1: Static game with population controls with cooperation.

In a static context, there exists a single Nash equilibrium in pure strategies in which both cities end up competing and do not cooperate.

### 3.3.2 Cooperation in a dynamic context

#### 1. Finite horizon

Consider now the case where cities are concerned not only for present outcomes, but for future land rents as well. Two scenarios can be differentiated. Assume first that competition among jurisdictions is to take place up to period  $T$  only, that is, for a limited number of years. Using backwards induction from the last period, it is found the usual result that, as in the static scenario, the set of pure strategies (*compete, compete*) make a Nash equilibrium despite it yields lower aggregate land rents. Thus, when the interaction only takes place for a finite number of periods, the resulting equilibrium strategy is again to compete and to deviate from the cooperative agreement.

#### 2. Infinite horizon

Consider now that cities interact for an infinite number of periods. In order to be able to calculate aggregate land rents in each possible situation, strategies must be defined for every contingency that may occur. The strategies considered are the following ones. Cities either

can cooperate or compete, the latter understood as fixing the population control as their respective reaction function suggests. It is assumed that cities follow the *trigger strategy*, that is, once any of them deviates from the agreed growth control, competition among them takes place in the subsequent periods. Thus, deviating leads to immediate gains in the current period, but implies renouncing to higher land rents in the remaining future due to the end of cooperation. How future land rents are discounted –together with the number of cities in competition– is one of the key factors among the conditions determining the equilibrium solution and the optimal strategy to be followed by each city.

Using the above assumption that cities compete from the moment that one of them does not cooperate, the present value of total land rents –PVTR– can be calculated for the different scenarios that can occur. Namely, the PVTR has been calculated in the following three instances:

- When cities always cooperate
- When cities always compete
- When one city deviates in the first period while the other one cooperates, and competition prevails from that period on.

Let  $PVTR^{coop}$  denote the present value of land rents when both cities cooperate;  $PVTR^{comp}$ , the present value of land rents when competition takes place;  $PVTR^{dev}$ , the present value of rents resulting from deviating in the first period and competing in the subsequent ones; and  $PVTR^{coop'}$ , the present value of rents when the city cooperates in the first period when the other one deviates, and both compete in the remaining periods. In figure 3.2 the dynamic game with infinite periods has been transformed to its single-period equivalent, where the payoffs represent the present value of the flow of land rents.

		City B	
		Cooperate	Compete
City A	Cooperate	$PVTR^{coop}, PVTR^{coop}$	$PVTR^{coop'}, PVTR^{dev}$
	Compete	$PVTR^{dev}, PVTR^{coop'}$	$PVTR^{comp}, PVTR^{comp}$

Figure 3.2: Infinite horizon dynamic game with population controls when cooperation is allowed.

The worst possible scenario for a city in terms of the aggregate total land rents occurs when it chooses the cooperative population control while the other city deviates in the first period, since  $PVTR^{coop'} < PVTR^{comp}$ .

Suppose first that city A chooses the population control according to its best response function, that is, it chooses the strategy “compete”. In this instance, the best strategy for city B is to compete as well, since doing so yields higher land rents. Similarly, that applies symmetrically to city A when it is city B the one that competes. Then, the set of strategies (*compete, compete*) constitute a Nash equilibrium.

Suppose now that city A cooperates. City B can choose its best strategy: either to cooperate as well, or to deviate from the cooperative solution and compete in all the remaining periods. To find out about the best strategy, we need to know which of the two leads to the highest payoff. Comparing  $PVTR^{coop}$  against  $PVTR^{dev}$ , it is found that the optimal solution depends on the value of the discount factor. Cooperating is optimal whenever

$$PVTR^{coop} > PVTR^{dev},$$

and comparing the two values of present value of land rents:

$$\begin{aligned} & \frac{t}{450k} [20\alpha^2 N_0^2 + 90\alpha N_0 N_1 + 45N_1^2 + 25\alpha N_0] \\ & + \frac{t}{r(1+r)450k} [20\alpha N_0^2 + 90\alpha N_0 N_1 + 45N_1^2 + 25\alpha N_0] > \\ & \frac{t}{450k} [23\alpha^2 N_0^2 + 96\alpha N_0 N_1 + 48N_1^2 + 25\alpha N_0^2] \\ & + \frac{t}{288r(1+r)k} [11\alpha^2 N_0^2 + 54\alpha N_0 N_1 + 27N_1^2 + 16\alpha N_0^2], \end{aligned} \quad (3.25)$$

which can be simplified to

$$\frac{(\alpha N_0 + N_1)^2}{160r(1+r)} > \frac{(\alpha N_0 + N_1)^2}{150}. \quad (3.26)$$

It results that the present value of land rents when cooperating exceeds that of deviating as long as

$$r < 0.589. \quad (3.27)$$



At the current level of interest rates, and according to the result above, if cities were to take decisions in an infinite horizon then it would be more profitable for them to cooperate, to choose smaller city sizes and as a result to set more stringent population controls. Then the set of pure strategies (*cooperate, cooperate*) would be a Nash equilibrium as well. However, if interaction among jurisdictions applies for a limited number of periods only, then the competitive solution prevails and population controls are somehow less restrictive. Although it is not such a strong assumption to consider that competition between jurisdictions may take place infinitely, it is however less likely that elected local government base their decisions in such a long term.

## 4 The effects of a tax on housing consumption

Other possible instruments to constrict city size are taxes that modify housing bid-rents of households, and consequently they somehow distort landowners' decisions of converting land from rural to urban. In this section the tax used will be one placed on housing. The rationale for introducing such a tax is to levy taxes that the community would be able to use to finance a public good, for instance. The main difference when utilizing a price instrument in this setting relates to the distributional consequences.

Consider now the case where a tax per unit of housing consumption, denoted by  $h^i$ ,  $0 < h \leq 1$ , is introduced in city  $i$ , whose residents now face an additional expense. This makes that their respective budget constraints change to:

$$Y_j = z_j + tr + R_j^i(r)s_j + h^i s_j, \quad (4.1)$$

or expressed in terms of the housing bid-rents:

$$R_j^i(r) = \frac{Y_j - tr - z_j}{s_j} - h^i. \quad (4.2)$$

The land bid-rent functions result then:

$$L_1^i(r) = k[Y_1 - tr - z_1 - h^i - P] \quad (4.3)$$

$$L_0^i(r) = k\left[\frac{Y_0 - tr - z_0}{\alpha} - h^i - P\right]. \quad (4.3')$$

## 4.1 Equilibrium values when one city uses taxes

The new land rent functions differ from the one in the market situation because of the new tax on housing,  $h^i$ . In equilibrium it must hold in any case that  $L_j^i = 0$ , independently of whether the city uses or not a tax. Assume first that only  $A$  introduces a tax  $h^A$ . Then  $\overline{r^B} = \overline{r^C} = \overline{r}$ , and since in equilibrium  $z_1$  is common to all cities, it results:

$$\overline{r} = \frac{\alpha N_0 + N_1}{2k} - \frac{\overline{r^A}}{2}. \quad (4.4)$$

Since  $L_1(\overline{r^A}) = 0$ , and using 3.1, the size of city  $A$  can be expressed in terms of the tax  $h^A$ :

$$\overline{r^A} = \frac{\alpha N_0 + N_1}{3k} - \frac{2h^A}{3t}. \quad (4.5)$$

There is a linear and negative relationship between the tax and city size. Introducing a tax on housing consumption also reduces housing rents and consequently land rents, what makes city  $A$  smaller. Now we can find the expressions for  $z_1$  and  $z_0$  in terms of the housing tax  $h^A$ :

$$z_1 = Y_1 - P - \frac{t}{3k}[\alpha N_0 + N_1] - \frac{h^A}{3}, \quad (4.6)$$

and

$$z_0 = Y_0 - \alpha P - \frac{\alpha t}{3k}[N_0 + N_1] - \frac{\alpha h^A}{3}. \quad (4.7)$$

It results again that  $\widehat{r^i} = \widehat{r} = \frac{\alpha N_0}{3k}$ . Examining the effect of  $h^A$  on  $z_0$  and  $z_1$ , it results that as the housing tax increases the consumption of  $z_0$  and  $z_1$  logically decreases, as does utility.

Both a certain population constraint and a tax on housing consumption that leads to the same city size have the same negative effect on households, and in either situation they reach identical utility levels of consumption of  $z$ . This is a logical result since both affect them in the same direction: the housing consumption tax acts as an additional expense for households, while the population control causes housing rents to increase. In both scenarios, the remaining income that can be dedicated to non-land goods shrinks. If the tax was on land instead of housing, then the city would become smaller as well,

residents would attain higher utility levels, but landowners activity in city  $A$  would become less profitable. But the distributional consequences differ. In the population control case, landowners of developed land receive higher land rents, while residents experience a reduction in their utility levels. When the housing consumption tax is used, then resident households also experience a comparable decrease in  $z$  and the utility level, but all landowners in city  $A$  lose too. Instead, the local authority benefits from all aggregate housing consumption taxes.

Which tax would the local government choose if its objective were to maximize the sum of aggregate taxes levied from residents in city  $A$ ? Since the tax affects the size of city  $A$  and also the number of households subject to the tax, the objective of the local authority would be

$$\max_h R^A = h^A k r^A = h^A k \left[ \frac{\alpha N_0 + N_1}{3k} - \frac{2h^A}{3t} \right]. \quad (4.8)$$

Maximizing the above expression yields the optimal value of  $h^{A*}$ , which is

$$h^{A*} = \frac{t}{4k} [\alpha N_0 + N_1]. \quad (4.9)$$

This optimal tax corresponds to a city size of

$$r^{A*} = \frac{1}{6k} [\alpha N_0 + N_1]. \quad (4.10)$$

After substituting 4.9 in the expressions of  $z_1$  and  $z_0$  in 4.6 and 4.7, it is obtained:

$$z_1(h^*) = Y_1 - P - \frac{5t}{12k} [\alpha N_0 + N_1], \quad (4.11)$$

and

$$z_0(h^*) = Y_0 - \alpha P - \frac{\alpha t}{12k} [5N_1 - (4 + \alpha)N_0]. \quad (4.12)$$

Both levels of private goods consumption are smaller compared to the market situation. The effects on  $z$  would be identical if directly using a population control leading to the city size achieved when using  $h_A^*$ . Residents lose in a similar way both with taxes and population controls. On the contrary, landowners gain with the introduction of the population control, but are worse off with the tax on housing that ultimately diminish land rents. Likewise, landowners of undeveloped land lose out with the tax. The benefits are for local communities which receive an income corresponding to aggregate taxes.

## 4.2 Equilibrium values when two cities use taxes

As with the population growth control case, consider now that all cities in the system except for the passive city  $C$  impose taxes on housing consumption. Thus,  $A$  and  $B$  enact taxes  $h^A$  and  $h^B$ . In order to maximize taxes levied, city  $A$  now must consider the behaviour of all other *active* cities, and so must city  $B$ . The expression for 3.8 in page 8 will determine the level of  $z_1$  in the system, common to the three cities. Applying the condition that  $L_1(\overline{r^A}) = L_1(\overline{r^B}) = 0$ ,  $z_1$  can be expressed exclusively in terms of the taxes applied by cities  $A$  and  $B$ , thus

$$z_1(h^A, h^B) = Y_1 - P - \frac{t}{3k}[\alpha N_0 + N_1] - \frac{1}{3}[h^A + h^B]. \quad (4.13)$$

And from the expressions in 2.6, it is found that

$$z_0(h^A, h^B) = Y_0 - \alpha P - \frac{\alpha t}{3k}[N_0 + N_1] - \frac{\alpha}{3}[h^A + h^B]. \quad (4.14)$$

The expression for  $\overline{r^A}$  can similarly be calculated:

$$\overline{r^A} = \frac{\alpha N_0 + N_1}{3k} + \frac{1}{3t}[h^B - 2h^A]. \quad (4.15)$$

The objective of city  $A$  will be again to maximize aggregate taxes levied, but now taking into account decisions taken by city  $B$ . The expression of  $\overline{r^A}$  in 4.15 is used. Thus, the maximization problem consists in:

$$\max_{h^A} R_h^A = h^A k \left[ \frac{\alpha N_0 + N_1}{3k} + \frac{1}{3t}(h^B - 2h^A) \right], \quad (4.16)$$

and the best response function for city  $A$  results in

$$h^A(h^B) = \frac{t(\alpha N_0 + N_1)}{4k} + \frac{h^B}{4}. \quad (4.17)$$

For city  $B$  the best response function could be analogously found:

$$h^B(h^A) = \frac{t(\alpha N_0 + N_1)}{4k} + \frac{h^A}{4}. \quad (4.18)$$

Notice that the sign of the partial derivatives is positive. Contrary to what happened in the population control game, now the best response functions

slope upward in the tax game. This means that taxes as growth control instruments act as strategic complements rather than substitutes. When  $B$  chooses a relatively high tax more households are diverted to the remaining cities in the system, including  $A$ , since the decision is made considering that  $h^A$  remains fixed, but not the population level in  $A$ . By imposing a higher  $h^A$  it is possible to increase the revenue levied from this diverted households.

The system with the best response functions for cities  $A$  and  $B$  is solved, and the equilibrium tax values corresponding to the simultaneous tax game result in:

$$h^{comp} = h^A = h^B = \frac{t}{3k}[\alpha N_0 + N_1]. \quad (4.19)$$

The tax revenue that arises from these equilibrium tax levels is

$$R_h(h^{comp}) = \frac{2t}{27k}[\alpha N_0 + N_1]^2 \quad (4.20)$$

for any of the cities enacting taxes. The resulting city size is

$$\bar{r}(h^{comp}) = \frac{2}{9k}[\alpha N_0 + N_1]. \quad (4.21)$$

This city size is larger than in the one controlling city case, and it is smaller compared to the population control game. However, comparisons must be taken carefully, since different objective functions have been used for the population and the tax control games. Alternatively, the total level of revenues should be compared<sup>3</sup>.

Other studies have already shown that price instruments lead to higher equilibrium populations (Helsley and Strange, 1995). As for the levels of  $z_0$  and  $z_1$ , which ultimately affect the levels of utility in the system, it results that

$$z_1(h^{comp}) = Y_1 - P - \frac{5t}{9k}[\alpha N_0 + N_1], \quad (4.22)$$

and

$$z_0(h^{comp}) = Y_0 - \alpha P - \frac{\alpha t}{9k}[5N_1 + (3 + 2\alpha)N_0]. \quad (4.23)$$

---

<sup>3</sup>This comparison of effects will be further explored in a upcoming section.

The consumption of non-land goods diminishes when introducing strategic interaction between cities, for both types of households, again compared to the outcome with a single controlling city. As the number of cities using taxes increases, the negative effects on  $z$  rise because it grows the number of households who are diverted from the controlling cities. This causes housing and land rents in the system to increase.

### 4.3 A cooperative framework with taxes

As in the population control scenario, we consider the possibility that cooperation between jurisdictions might take place when enacting taxes. To simplify, and since no a priori differences between cities A and B exist, one would expect the implementation of symmetrical tax rates<sup>4</sup>. This permits to express aggregate revenues from taxes for any of the active cities as:

$$\max_{h^{coop}} R_h(h^{coop}) = h^{coop}k\left[\frac{\alpha N_0 + N_1}{3k} - \frac{1}{3t}h^{coop}\right]. \quad (4.24)$$

It is found that the optimal tax under cooperation is

$$h^{coop} = \frac{t}{2k}[\alpha N_0 + N_1], \quad (4.25)$$

tax size that exceeds the Nash equilibrium tax level,  $h^{coop} > h^{comp}$ . Accordingly, the resulting city size for the controlling cities is

$$\bar{r}(h^{coop}) = \frac{1}{6k}[\alpha N_0 + N_1], \quad (4.26)$$

which is smaller compared to the outcome obtained in the competition scenario.

Let  $R_h(h^{coop})$  denote the tax revenue when both cities cooperate. Likewise,  $R_h(h^{comp})$  in equation 4.20 denoted the revenue with competition. It can be observed that  $R_h(h^{coop}) > R_h(h^{comp})$ , that is, tax revenues are larger under the cooperative framework. Assuming that one city enacts the cooperative tax, the best tax level for the other city can be calculated by substituting in the city's best response function. Let  $h^{dev}$  denote the tax chosen when deviating. Then,

$$h^{dev} = \frac{3t}{8k}[\alpha N_0 + N_1], \quad (4.27)$$

---

<sup>4</sup>Alternatively, it could be accepted a different agreement on taxes leading to this maximum level of tax revenues, followed by an egalitarian share-out of taxes levied

that is, the city that deviates from the agreement gains from imposing a lower tax rate. The gains from the attraction of a larger number of residents offset the loss associated to collect a smaller tax per head of population. The overall tax revenue for the city that breaks the cooperation agreement is

$$R_h(h^{dev}) = \frac{3t}{32k}[\alpha N_0 + N_1]^2, \quad (4.28)$$

while the city that enacts  $h^{coop}$  obtains a diminished tax revenue of

$$R_h(h^{coop'}) = \frac{t}{16k}[\alpha N_0 + N_1]^2, \quad (4.29)$$

where  $R_h(h^{coop'})$  denotes overall tax revenue for the city that fixes  $h^{coop}$ . If only revenues from a single period are considered, then the highest revenues are obtained when cities deviate from the cooperation agreement, while the other one maintains the cooperative tax  $h^{coop}$ . Thus,

$$R_h(h^{dev}) > R_h(h^{coop}) > R_h(h^{comp}) > R_h(h^{coop'}) \quad (4.30)$$

### 4.3.1 Equilibrium in a static context

Consider the case where the effects of the introduction of taxes are going to be realized for a single period only. Then, as it happened in the population control game, the equilibrium solution is to compete and to enact  $h^{comp}$  as tax sizes. Although communities would be able to attain higher revenues if they maintained the cooperative tax level, cooperating is not an equilibrium. Keeping in mind the comparison of revenues as shown in equation 4.30, the

		City B	
		Cooperate	Compete
City A	Cooperate	$R_h(h^{coop}), R_h(h^{coop})$	$R_h(h^{coop'}), R_h(h^{dev})$
	Compete	$R_h(h^{dev}), R_h(h^{coop'})$	$R_h(h^{comp}), R_h(h^{comp})$

Figure 4.1: Static game with taxes when allowing for cooperation.

equilibrium solution can be found when solving the static game depicted in table 4.1. Given a certain strategy of the rival community, and in the search of the largest payoffs, it is found that the only Nash equilibrium is the one in which cities compete, and they achieve an identical tax revenue of  $R(h^{comp})$ .

### 4.3.2 Equilibrium in a dynamic context

Two scenarios, the finite and infinite horizon games are considered in this subsection. As in the population control case, cooperation would be only the resulting equilibrium when considering that the game lasts infinite periods.

#### 1. Finite horizon

When the interaction between jurisdictions takes place for a limited number of periods, again the equilibrium strategy for any of the cities is to compete. To solve this game, backwards induction is used, and in every single period the equilibrium solution is to compete.

#### 2. Infinite horizon

If the game between jurisdictions is to take place during infinite periods, then cooperation is a plausible equilibrium solution. The calculus of the revenue obtained in every possible scenario is needed. Thus it has been calculated the respective present value of tax revenues when the two cities cooperate  $-PVR_h(h^{coop})-$ ; when the two cities compete  $-PVR_h(h^{comp})-$ ; and when one city cooperates while the other one deviates from the cooperation agreement  $-PVR_h(h^{coop'})$  and  $PVR_h(h^{dev})$ , respectively.

In each instance, the expressions for aggregate tax collections result as shown below. Aggregate revenues when one city cooperates and the other one deviates have been calculated under the assumption that the trigger strategy applies, as in the population control game. The results are as follows:

$$R_h(h^{coop}) = \frac{t}{12k}[\alpha N_0 + N_1]^2 + \frac{t}{12kr(1+r)}[\alpha N_0 + N_1]^2 \quad (4.31)$$

$$R_h(h^{comp}) = \frac{2t}{27k}[\alpha N_0 + N_1]^2 + \frac{2t}{27kr(1+r)}[\alpha N_0 + N_1]^2 \quad (4.32)$$

$$R_h(h^{dev}) = \frac{3t}{32k}[\alpha N_0 + N_1]^2 + \frac{2t}{27kr(1+r)}[\alpha N_0 + N_1]^2 \quad (4.33)$$

$$R_h(h^{coop'}) = \frac{t}{16k}[\alpha N_0 + N_1]^2 + \frac{2t}{27k}[\alpha N_0 + N_1]^2 \quad (4.34)$$



To find the equilibrium solution, one must compare aggregate tax revenues if cooperation prevails and if cities deviate and choose  $h^{dev}$ . The latter allows for a greater revenue in the first period, but that leads to the smaller competition revenue levels for all the periods on. Once

		City B	
		Cooperate	Compete
City A	Cooperate	$PVR_h(h^{coop}), PVR_h(h^{coop})$	$PVR_h(h^{coop'}), PVR_h(h^{dev})$
	Compete	$PVR_h(h^{dev}), PVR_h(h^{coop'})$	$PVR_h(h^{comp}), PVR_h(h^{comp})$

Figure 4.2: Infinite horizon dynamic game with taxes, when allowing for cooperation.

the cooperation agreement has been abandoned by one of the cities, competing becomes the Nash equilibrium strategy. The cooperative solution is a Nash equilibrium as well if the present value of revenues under cooperation exceeds the present value of revenues when deviating, that is

$$R_h(h^{coop}) > R_h(h^{dev}), \quad (4.35)$$

or

$$\begin{aligned} \frac{t}{12k}[\alpha N_0 + N_1]^2 + \frac{t}{12kr(1+r)}[\alpha N_0 + N_1]^2 > \\ \frac{3t}{32k}[\alpha N_0 + N_1]^2 + \frac{2t}{27kr(1+r)}[\alpha N_0 + N_1]^2 \end{aligned} \quad (4.36)$$

Operating the expression above and solving for  $r$  it results that cooperation is the equilibrium solution as long as the interest rate is

$$r < 0,875. \quad (4.37)$$

Again, with current interest rates, cooperation would be the equilibrium strategy for local jurisdictions when the game lasts infinite periods.

## 5 Comparison of results with population controls and taxes

We have so far assumed that the housing tax and the population control were endogenous in the sense that they maximize total taxes levied by local communities. In this section aggregate taxes levied are compared depending on the instrument used. Because households' utility levels do not depend upon any local public good or urban amenity, the expenditure side of the tax collection is being ignored. It can be likewise argued that the positive effect on households' utility of this expenditure would be the same for a constant level of tax revenues, and then only the negative effects associated to the particular source of the fiscal revenue matter.

To compare the different effects of using population controls or taxes for collection purposes, we make the following assumption. Taxes on housing directly yield a certain amount of tax revenue. As for the population control effect on fiscal revenues, two extreme scenarios can be considered. Firstly, it could be the case that only increased land rents were taxed, for instance if the whole increase in land rents was captured by the local government. The second possibility consists in assuming that the local community imposes a certain  $p\%$  tax on total land rents, not only on value increases. With a  $p=100\%$ , the local community would appropriate all land rents. Such an extreme tax on land rents creates the problem of landowners losing the incentive to efficiently allocate each plot of land to the highest bidder.

The comparison of tax revenues under each instrument results easier if representing each revenue level against the associated city size  $\overline{r^A}$ , as in figure 5.1. The graph plots the revenue size associated to each city size, under three different scenarios: when cities compete with taxes; when cities compete with population controls and the tax revenue equals the increased land rents; and when cities compete with population controls and the tax revenue equals *all* land rents. In any case it is also assumed that cities choose identical housing taxes, or identical population controls, so that only symmetric solutions are considered.

The relationship between land rents –or increased land rents– and city size is straightforward, by rearranging the general expression of land rents in equation 3.4. As for the relationship between tax revenues and city size, it can be easily obtained by combining the expression of tax revenues in equation 4.8 with the expression in equation 4.5 in page 16, that inversely relates the

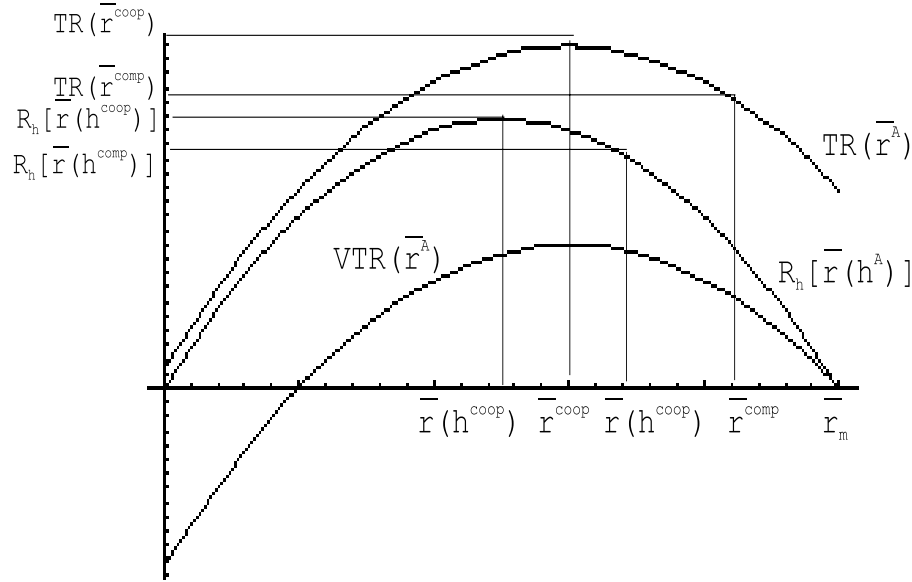


Figure 5.1: Comparison of tax revenues with population controls and housing taxes

tax level  $h^A$  and city size  $\bar{r}^A$ . Each tax level is uniquely associated to a certain city size. As it can be observed, the three revenue curves follow the Laffer type shape. Thus, a small city size can represent either the utilization of a too stringent population control or the result of a relatively high tax on housing consumption. A smaller city is associated then either to a large increase in land rents or to a higher housing tax. Both facts provoke a greater per capita revenue, but a reduction in the base of the revenue due to the fact that less residents remain in the city in equilibrium. Several city sizes have been highlighted:  $\bar{r}_m$ , which represents the city size corresponding to the market situation;  $\bar{r}^{comp}$ , the equilibrium city size when competing with population controls;  $\bar{r}(h^{comp})$ , the arising city size when competing with taxes;  $\bar{r}^{coop}$ , the city size when cities agree on population controls; and  $\bar{r}(h^{coop})$ , the resulting city size when cities cooperate to fix their taxes.

A city size of  $\bar{r}_m$  corresponds to a situation where there is no population control or a tax  $h^A = 0$ , and as a result  $R_h = 0$  and  $\Delta TR^A = 0$ . Total land rents equal the value market, that is  $TR^m$ .

When the increases in land rent values due to the introduction of the population control are fully taxed, taxes are always superior to population

controls because a fixed revenue level can be achieved at a smaller cost in terms of the decrease in residents' utility. The optimal city size when maximizing increased total land rents  $\Delta TR^A$  is  $\bar{r}^A = \frac{1}{5k}[\alpha N_0 + N_1]$  –the same city size that also maximizes aggregate land rents  $TR^A$ . The revenues arising from the implementation of a tax on housing leading to the same city size are greater. Alternatively, the revenue obtained with this city size,  $\Delta TR^A = \frac{t}{144k}[\alpha N_0 + N_1]^2$ , could be achieved with a tax level leading to a greater city size –and as a result, with a reduced loss in residents' utility. For values of the city size greater than the optimal level  $\bar{r}^A$  the diverting of population through a direct population control provokes that total revenues begin to decline, up to the city size  $\bar{r}^A = \frac{1}{6k}(\alpha N_0 + N_1)$ , that implies again that  $\Delta TR^A = 0$ .

Secondly, consider that a tax levies total land rents, and not only land rent rises. Under this scenario the comparison between taxes and population favors the population control instrument, since total land rents are always superior to taxes in terms of total revenue for identical city sizes. Under this total confiscation of land rents it arises the problem that landowners have no incentive to efficiently allocate their land.

There exists an intermediate tax rate  $p$  that could be applied on total land rents, that would lead to identical outcomes in terms of tax revenues. Analytically, this  $p$  tax rate can be expressed in terms of the parameters, being

$$p = \frac{18k^2(\alpha N_0 + N_1)^2}{-45\alpha^2 N_0^2 + 16\alpha k^2 N_0^2 + 56\alpha^2 k^2 N_0^2 - 90\alpha N_0 N_1 + 144\alpha k^2 N_0 N_1 - 45N_1^2 + 72k^2 N_1^2} \quad (5.38)$$

## 6 Conclusions

This paper provides an extension to the scarce urban economics literature undertaking the analysis of urban regulations as the result of the strategic interaction among local jurisdictions. Some results previously cited have already be found, as would be the distinct nature of quantity and price instruments as strategic variables to manage urban growth. Thus, population controls are strategic substitutes while taxes act as strategic substitutes. Likewise, generalizing the model for  $n$  cities, it could be shown that the decrease in the utilities of residents increases with the number of cities using controls.

Measured in terms of resident's utility, competing with population controls is desirable because it leads to relatively greater city sizes, and consequently, to smaller negative impacts on utilities. If measured in terms of total revenues, population controls are superior to taxes only when all land rents are confiscated, but inferior when only increased land rents constitute the tax revenue.

It has also been considered the case where competition takes place not only in a single period, but repeatedly. When the interaction between jurisdictions takes place in a finite horizon, the equilibrium strategy for every city is to compete. In a infinite period horizon, though, cooperation becomes the equilibrium strategy for active cities for "normal" levels of interest rates.

This work is still in a preliminary phase. The model used incorporates variables which have not been exploited in the present analysis, but that will hopefully constitute the focus of further research. For instance, one serious shortcoming is that externalities have not been considered, while their presence makes one of the most alleged reasons to justify urban land controls from an economics perspective. The model seems flexible enough so as to easily incorporate environmental externalities affecting households welfare, both in the form of density levels  $-k$ —and of the costs of urban growth and the consequent loss of open spaces. Similarly, two different types of households have been differentiated attending to their income levels. Additional attention and a more careful analysis should be devoted to the distributional consequences of planning instruments.

## References

- Anas, A., Arnott, R. and Small, K. A. (1998). Urban spatial structure, *Journal of Economic Literature* **XXXVI**(3): 1426–1464.
- Brueckner, J. K. (1990). Growth controls and land values in an open city, *Land Economics* **66**(3): 237–248.
- Brueckner, J. K. (1997). Infraestructure financing and urban development: The economics of impact fees, *Journal of Public Economics* **66**(3): 383–407.

- Brueckner, J. K. (1998). Testing for strategic interaction among local governments: The case of growth controls, *Journal of Urban Economics* **44**(3): 438–467.
- Brueckner, J. K. (2001). Urban Sprawl: Lessons from Urban Economics, *Brookings-Wharton Papers on Urban Affairs* (forthcoming) .
- Engle, R., Navarro, P. and Carson, R. (1992). On the theory of growth controls, *Journal of Urban Economics* **32**(3): 269–283.
- Fischel, W. A. (1990). Introduction: Four maxims for research on land-use controls, *Land Economics* **66**(3): 229–236.
- Helsley, R. W. and Strange, W. C. (1995). Strategic growth controls, *Regional Science and Urban Economics* **25**(4): 435–460.