The End of Euclidean Geometry or Its Alternate Uses in Computer Design

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ABSTRACT

This paper presents two methods of computer-based architectural design. Computer use requires procedures that enable the achievement of user objectives. However, Computer Aided Design (CAD) has been frequently used as an electronic drawing board.

KEYWORDS: architectural computer design, fractal geometry, non-planar projection.

The most important tool in the field of architectural computer design is the software known as CAD, or Computer Aided Design. However CAD has been used, not infrequently, for drawing the graphical expression of architectural projects much more than for the creation of a design. Therefore, CAD has been identified as an electronic drawing board. By moving the mouse across the screen one is effectively doing the same as drawing lines on paper with pencil.

The geometric primitives that the system can utilize induce or generate forms related to the vocabulary of rationalist architecture.

Euclidean Geometry, a projection on a flat surface, seems absent in today's vanguard architecture; therefore some authors comment on the end of Euclidean geometry in the field of computer design. In fact, many of the new forms in some contemporary architectural works surpass the limits of that geometry, either by freedom in modeling or by adopting non-planar projection. Rather than the ultimate death of traditional geometry concepts we might be experiencing an unusual change in the way it is used.

In the last century we saw the emergence of an architecture that had abandoned the principles of rationalism, functionalism, or the tendency of organic architecture. The development of new ideas in many different fields, but especially in science and its consequent reflection in technology has resulted in new ways of thinking and of doing things. In the field of architecture technological advances in electronics and computers offered more than the freedom to devise new methods of realizing new dreams, they also secured the tools to design and implement a formal expression of new ideas.

Even in the complex forms of architecture, graphical representation such as planes, elevations, and sections are done by using Euclidean geometry. We see Euclidean geometry and any other non-planar projection as complementary and not necessarily as antagonists. In modern architecture non-planar geometry accounts for design but planar projection performs its specific purpose.

However these two different types of geometry are seen through his contradictory relationship. Examining Eisenman's architecture of the Biocentre at the J. W. Goethe University of Frankfurt it has been observed that they "meshed fractal geometry with Euclidean geometry, 'infecting' one geometry . . . with an equally available one" (Ostwald, 2001).

There are different reasons for the negative reactions to postmodernism. One, and perhaps the most important, is its lack of rationality, as if we were in a historical phase that missed the Age of Enlightenment. This is because, not as an observation but as a world view artists, thinkers and historians claim that the increase in skepticism, relativism and subjective thought, and the growing distrust of reason, are the basic features of postmodern culture today.

As a totally idealistic focus, we critically identified with the myth of Plato's cave. In architecture the question of feasibility or constructiveness is often relegated to the background. Specifically, faced with the shadow of an object that is outside of the cave, out of sight, we do not know with what object a shadow matches. This is a metaphorical allusion. What we consider important is whether there are methods that can establish a uniform understanding of the relationship between the object and its shadow. Is there any possibility of identifying methods and procedures to specifically and methodically execute architectural forms that are more free, creative and independent of the traditional patterns of so-called modern architecture.

These innovations have not only generated a behavior that does not integrate the already established standards or principles, they have all but guaranteed such behavior. Diverse styles have appeared, although none of them has become a *school* of architecture. Today the use of the computer is very common in architectural production, which allows freedom of creativity. However, the use of new technology implies the need for knowledge and expertise in using it.

Archimede's statement "give me a place to stand and I will move the earth" should be remembered. New technology is available as the support needed to lift the world. Nevertheless, without a lever nothing can be done and the lever must be made of a strong material.

We understand the computer as a place or point of support. The lever is found in the way we work with it, the procedures and methods we use, and the knowledge, creativity, and savoir-faire we apply to it. In this paper we address two non-traditional procedures for modeling 3D. The first is a procedure based on fractal geometry. The second is based on a free reference to Mercator's transverse projection of the surface of the earth.

We believe that in the context of this paper, this explication of methods and procedures can be metaphorically compared to the resumption of the Age of Enlightenment—chemistry instead of alchemy.

The Use of Fractal Geometry

It has been claimed that the relationship between fractal geometry and architecture can be found in many buildings "including various medieval castles, baroque churches, Hindu temples and works of Frank Lloyd Wright or Louis Sullivan [...] are not considered to be a consciously created fractal designs even if they display an intuitive grasp of fractal geometry" (Ostwald, 2001). Another author wrote "We can start the research of the buildings' self-similarity by the year 1104. In fact in the cathedral of Anagni (Italy) there is a floor which is adorned with dozens of mosaics, each in the form of a Sierpinski gasket fractal" (Salla, n. d.).

In the scientific community, more precisely in the mathematical field, fractal geometry was explained in the work of Benoit Mandelbrot *Les objets fractals, forme, hasard et dimension* (Mandelbrot, 1975). His popularity as a kind of denier cris was disseminated in James Gleick's *Chaos: Making a New Science* (Gleick, 1981). Around the same time book was published software that generated many different types of fractal images following different algorithms or methods was made available to the public.

Two main concepts or ideas were related to the geometry of fractals: the structure of chaos in complex systems where the rules of performance are not determined, and the beauty of the fractal images. Many books illustrate the second aspect. As a metaphor we can say that it identifies a trend. Any image which shows complexity in expression, symmetry, repetition of the same shape at different scales, or resemblance to natural forms, for instance, is understood as fractal.

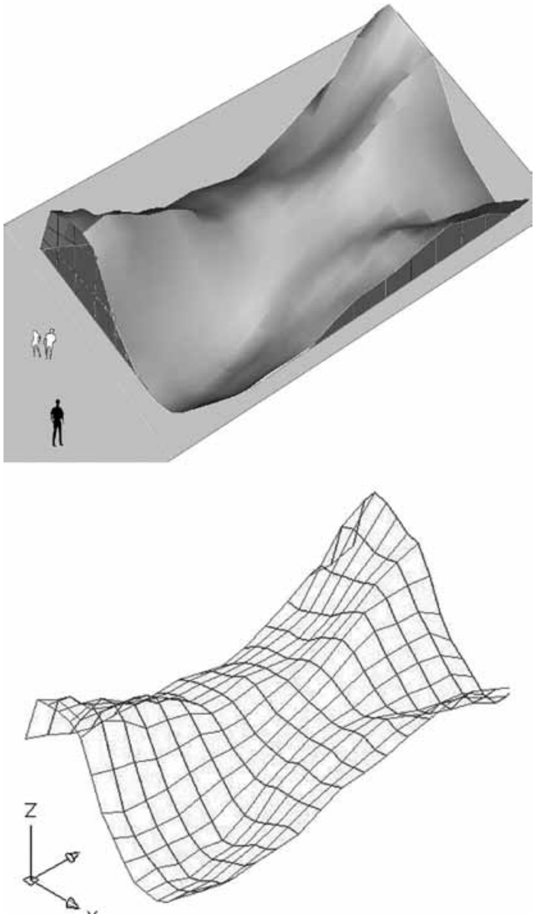
A fractal has its mathematical expression, and that is from where it derives. In the gap between a complex and symbolic language and another which is analog and intuitive, there is greater identification with the second; thus the popularity of fractal images.

In this paper the use of fractal geometry in computer design is more related to the adoption of methods for computer processing than to the analog tools found in CAD systems such as the mirror, the scale, and rotation etc. An experiment was performed using the James Gleick's Chaos software (Dantas, 1992), and through an exploratory process we came to a form that is a model for a theater (Fig. 1).

Despite the complexity of iterative functions with complex numbers, that is, a "complex number has two constituent pieces, the real part x and the imaginary part y" (Peitgen & Saupe, 1987), we observed that this function sweeps through a loop of a matrix where two-dimensional coordinates representing pixel position in the computer video giving each one a color whose code number is generated by an iterative function through a "process of repeatedly taking the results of a function and feeding them back into the same function" (Oliver, 1992). The ACAD system colors can be represented by a single decimal number, which facilitates the adoption of this number as the height of points in a Cartesian system of 3D projection

The derived procedure is to take this code color number as the coordinate of the height of points in a 3D representation. Through a VBA computer program running in ACAD (Cottingham, 2001), a fractal image is obtained as part of a Mandelbrot set (Stevens, 1989), and colors are controlled in order to have a desired amount of them (the larger the number the more complex the form of architecture).

Placing a 3D surface mesh over a fractal image and determining the number of mesh nodes, we assigned a value to the



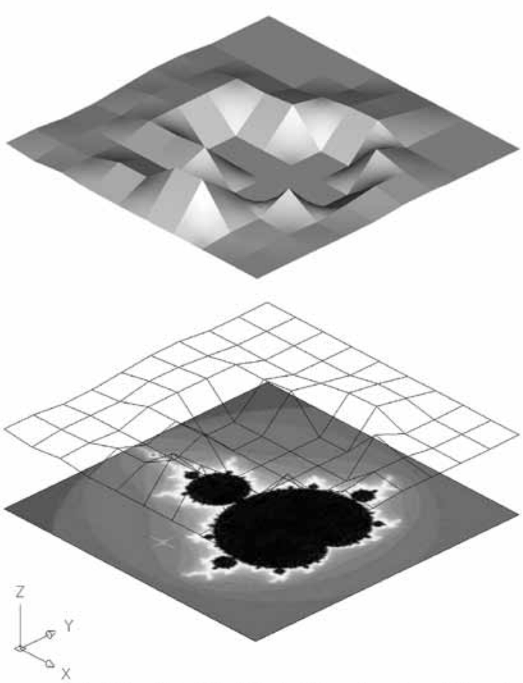


Figure 1. Sketch for a theater.

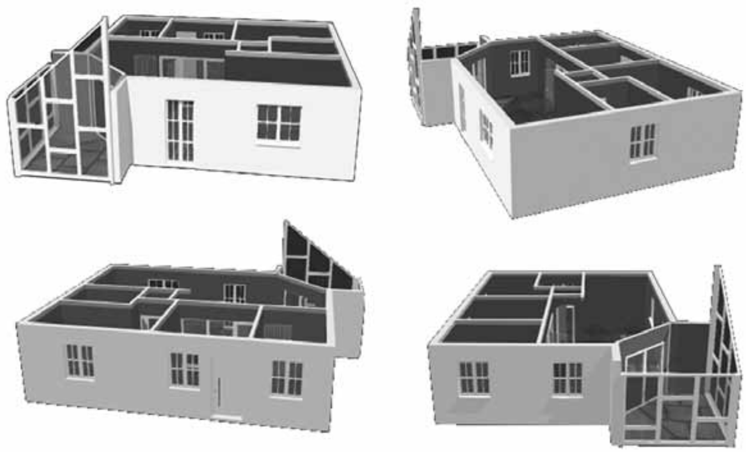


Figure 2. Mandelbrot fractal, wire 3D mesh, rendered 3D mesh

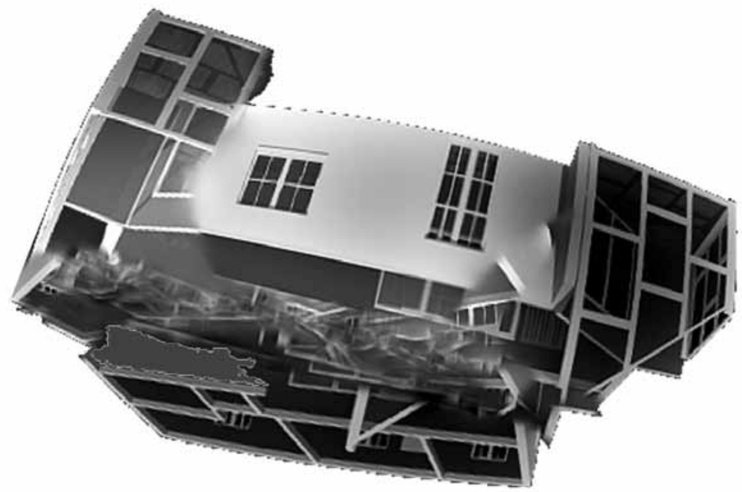


Figure 3. Different views of a 3D model

nodes of the color code as its *z* coordinate. We used a gray-scale simplifying a numeric code for colors. In doing so we obtained a 3D surface as represented in figure 2 (Fig. 2). The procedure of practically employing the concept of fractals for an architectural form follows these steps:

- Define the minimum and maximum coordinates of a four-sided polygon of (xmin, xmax, ymin, ymax) to match the dimensions of the building to be designed.
- Set the variable *k* as color and its maximum desired value (kmax), in order to achieve a small scale that has reduced color variation (in the fractal image) and acceptable values for the *z* coordinates of the range of the 3D shape height.
- Define the variables *sx* and *sy* as steps in the for loops, in *x* and in *y*, so that sx can be equal or different to *sy*. This step will result in the distance between the *x* axes (*xn*+1 – *xn*) = *sx* and *y* axes, (*yn*+1 – *yn*) = *sy*.
- Perform the calculation of the iterative function obtaining the value *k* (color code or *z* coordinate) of point x_o, y_o, z_o to *xn*, *yn*, *zn*.

- Perform the drawing of 3D poly-lines for each step of the for-next loop, taking the values *xn, yn, zn* for the poly-lines nodes
- Up to this point all transactions are processed by computer through a VBA code running in ACAD. From there we use a personal interface to draw a 3D mesh. We can choose a mesh with more discrete sections defined by the calculated points or a mesh defined by the path of each 3D poly-line.

We should note that the representation of the fractal image with the 3D shape is not necessary. This representation in this paper is merely illustrative.

The Procedure of Projection in a Cylinder

The procedure proposed uses the technique of panoramic photography that is common today. The first step is to get different views (in a total of 360 degrees) of a 3D model of a building (Fig. 3). Afterward, the images are treated with software designed to generate panoramic images. By joining all the images with the superposition of common or control points and projecting the resulting image on the surface of a cylinder the result is the image in found in figure 4 (Fig. 4).

Usually the images such as those of a landscape or the inside of a building are projected onto the inner surface of a cylinder after the stitching process because the observer is within the landscape or the building. But if the pictures are taken outside the building, its projection on the inner surface of the cylinder produces a deformation that can be compared to the inverse process of Mercator's UTM map projection.

A comparison can be made. If a panoramic image of the interior of a building is achieved through several photos taken by keeping the camera fixed on its vertical axis but rotating the camera around this axis in multiple steps until the camera rotates 360 degrees, and then the images are stitched together through the control points, by putting them all together we get large 360 degree image. But if photographs are taken of a 3D model of a building by placing the camera on the outside of the model, in order to create 360 degree image the model must be rotated because if the point of view of the camera is shifted and the model moves out of the viewing angle the control points obtained generate overlapping images instead of generating images that complement panoramically.

Conclusion

This paper aims to present methodological approaches to computer use. Both methods have dealt with two dichotomies. The first is related to the antagonism between non-planar projections and Euclidean geometry; the second is between the concept of drawing and the concept of designing.

Before its construction, a complex and non-planar architectural model necessarily has a representation based on Euclidean geometry, that is, a set of drawings that represent the complex continuity of designed shapes through discrete sections to offer a better understanding of the model. This flat representation is often required when performing structural calculations for a building. Even when using advanced technologies, freely shaped clay is covertly represented by the planar and sectioned projection in order to develop numerical codes for the numerical control apparatus that materializes the design.

In the second method an architectural 3D model is initially represented by a raster image. Once processed by the computer to obtain its vector expression, this image goes through an adjustment that is more drawing than design. The phenomenon observed in this second method, then, is that the use of CAD as a computer aided design does not eliminate its use as a drawing aid so the problem is its use as only an electronic drawing board. If so, then one can say that Euclidean geometry is still alive.

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