OPTIMUM WATER ALLOCATION IN A MULTIPURPOSE RESERVOIR WITH HOURLY VARYING HYDROELECTRIC TARIFF AND SEASONALLY VARYING AGRICULTURAL PRODUCTION RESPONSE

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ABSTRACT

Hydropower plant reservoirs are quite widespread in most countries and play an important role in water management. Its multipurpose character, combined with the natural scarcity of water resources, frequently leads to complex water management problems. Optimization models can be a useful tool for decision makers, especially if conflicting water uses are involved.

In this paper we will describe an optimization model that has been developed to compute the optimal water allocation for a multipurpose hydropower plant reservoir subject to: flood control; downstream seasonal water releases; hourly varying hydroelectric tariff, and seasonally varying agriculture production. The interesting aspects of this model include the profusion of water uses, the nonlinear character of the problem, and the time step harmonization, given that the evaluation of water deficit impact on agricultural production requires much longer time step periods than the water deficit impact on hydropower production, which can be computed on an hourly basis. After formulation the problem is solved using nonlinear programming. The model is described and its use discussed.

KEY WORDS

multipurpose reservoir, agriculture production function, hydroelectric production, nonlinear optimization.

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1. INTRODUCTION

Reservoir dams are hydraulic structures widely found in many countries. Its multipurpose character, combined with the natural scarcity of water resources, frequently leads to complex water management problems. This can be the case when multipurpose reservoirs are committed to the main tasks of hydroelectric production and agricultural irrigation by diverting water upstream. From the planning point of view, water sharing should be established taking into account the maximization of global benefit: hydroelectric production, and agriculture production.

There is a remarkable difference between hydroelectric production remuneration and agricultural production remuneration. Usually, hydroelectric remuneration depends on the amount of energy produced in each of the day's tariff periods. The seasonal variation of the hydroelectric tariff is usually small. Agricultural remuneration depends on the crop, which can only be known after several months of agricultural irrigation. As this paper shows, we can see that the decision process relative to the mutually exclusive use of the water must be able to deal with these two aspects, which have quite different time steps.

There has been great deal of research on reservoir management in the last three decades. Extensive literature reviews can be found in Simonovic (1992), Wurbs (1993), Labadie (2004) and Revelle (1999). Cunha (2003) discusses the operation research methods applied to solve problems in this type of problems. They range from the classical methods to more recent heuristic methods (Neelakantam and Pundirakanthan 2002). When building the decision model there are two aspects that deserve a particular attention: the agricultural production function and the hydroelectric tariff.

The production of a given type of plant depends on many different factors, particularly the amount of water available and its distribution during the vegetative life cycle. In the decision model presented here, an agricultural production function is used in which water is a decision variable and it is supposed that no other factors limit the production.

An analysis of the literature shows that there are two types of approach to building agricultural production functions. The first type includes models that use a physiological approach, where the development results from a complex interaction between various physiological aspects (stomatic behaviour, photosynthesis, etc.), related to the amount of water available for irrigation. Usually they are not well systematized and are built for specific case studies. Hsiao et al. (1976) emphasize the difficulty of building a model of this type, when all the aspects contributing to plant development have to be considered. The most widespread models, like that devised by Doorenbos and Kassan (1979), used in this work, employ evapotranspiration for such purposes:

$$\left(1-\frac{Ya_i}{Ym_i}\right)=Ky_i\left(1-\frac{ETa_i}{ETm_i}\right) \tag{1}$$

Ya_i=actual production in period i; Ym_i=maximal production (when no factor limits production) in period i; Ky_i=yield response coefficient in period i; ETa_i=actual evapotranspiration in period i; ETm_i=maximal evapotranspiration in period i (if there is not an irrigation deficit).

It is important to point out that the final production depends not only on the total irrigation occurring during the vegetative life cycle, but also on the allocation of the available water in the different periods of this life cycle. Two approaches have been considered: a first approach that considers additive effects of the water deficits (Jensen 1968) and a second one that considers multiplicative effects.

The latter appears to be more realistic since it determines the development in each period, accounting for the conditions observed in the previous periods. The model adopted in this work follows this last approach and is taken from Bowen and Young (1985):

$$\frac{Ya}{Ym} = \prod_{i=1}^{N} \left(\frac{Ya_i}{Ym_i} \right) \tag{2}$$

Ya=actual production ; *Ym*=maximal production (when no factor limits production).

Figure 1 was obtained was obtained by minimizing and maximizing agricultural production, as a function of the sharing of the total irrigation through the three vegetative periods of the given plant. The agricultural production function was adopted in the examples that we will present in Section 3. Indicative value of the agricultural production (€/m^2) are also shown.

In many countries the hydroelectric tariff varies through out the day. In Portugal the independent hydroelectric producers are paid according to a complex tariff. This tariff takes into account the CO_2 emissions that are avoided, the electric energy produced in peak and average consumption hours, the electric energy produced in low consumption hours, as well as the average monthly electric power produced.

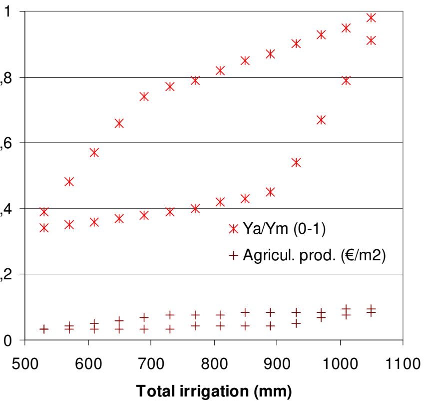

Figure 1: Final agricultural production (*Ya/Ym*) versus most efficient (upper values) and most inefficient (lower values) irrigation

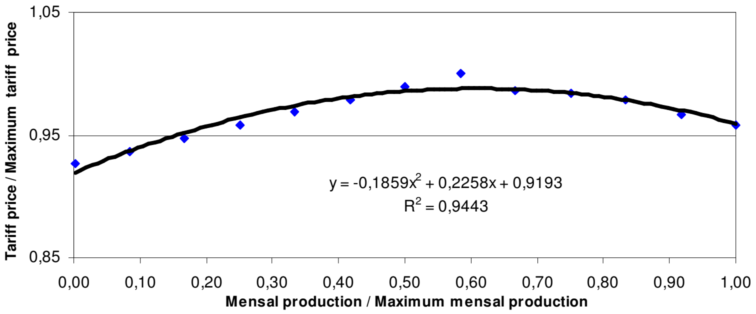

Figure 2: Variation of the hydroelectric production tariff with the monthly production

In Figure 2 we present the variation of the unit price of hydroelectric independent production with the monthly production, computed taking into account current Portuguese law, and considering that pondage capacity is used to transfer inflows from low consumption hours to peak and average consumption hours.

In the examples that we will present in Section 3, the second order polynomial, shown in Figure 2, was adopted to represent the hydroelectric production tariff.

2. FORMULATION OF THE PROBLEM

The problem that we want to analyze is now briefly described: for each of the 12 fortnights: how much water shall be allocated to agriculture, to hydroelectric production and released downstream in order to achieve maximum global remuneration in the 6 month crop period.

The objective function is:

$$\max \ R = \prod_{i=1}^{N} \frac{Ya_i}{Ym_i} Ym \cdot A \cdot Py + \sum_{i=1}^{N} \eta_i \cdot \gamma \cdot QT_i \cdot H_i \cdot Pe_i \tag{3}$$

R=global remuneration; N=number of time steps; i=integer that represents the time step period; A=area of the agricultural plant; Py=unitary price of the agricultural production; η_i=overall efficiency of the hydropower plant during period i; γ=constant that depends on the water density; QT_i=volume of water to be used by the turbines during period i; H_i=Gross head during period i; Pe_i=tariff price of the hydroelectric production during period i.

Figure 3 illustrates the inflows and outflows involved in this problem.

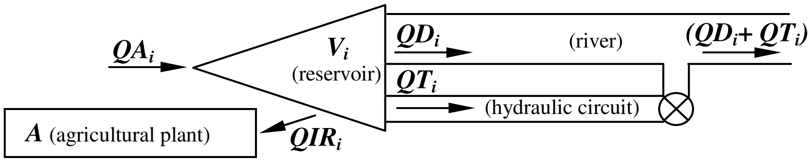

Figure 3: Schematic layout of inflows and outflows.

The agricultural production function is given by equation (1) already presented. The actual evapotranspiration depends on the water available in the soil, including the effective precipitation P_i, the net irrigation I_i and the water present in the soil before the occurrence of the latter two W_i. The evapotranspiration occurrence is intrinsically related to the easily usable fraction of the total water quantity. If the evapotranspiration occurs when the soil contains an amount of water corresponding to the easily usable fraction, therefore $Eta_i=Etm_i$. If the available water in the soil is below the easily usable fraction the evapotranspiration becomes a very complex process very difficult to model. For operational purposes, in this paper, it is used a simplified model (Cunha et al. 1993) based on Doorenbos e Kassan (1979). In this model, the evapotranspiration is obtained through a function of the soil water index ASI_i:

$$ETa_i = (ca_i + cb_i ASI_i) nd_i \tag{4}$$

ca_i, cb_i = coefficients of the linear regression model; nd_i=number of days of period i; ASI_i=soil water index (represents the fraction of the period where $Eta_i=Etm_i$).

ASI_i is determined by:

$$ASI_i = \frac{\left[P_i + W_i + I_i - (1-p)DSa\right]}{ETm_i} \tag{5}$$

with

$$W_i = W_{i-1} + P_{i-1} + I_{i-1} - ETa_{i-1} \tag{6}$$

$$I_i = Ef \cdot Ia_i \tag{7}$$

Ia_i=irrigation (diverted from the reservoir) in the period i; Ef=efficiency of the irrigation system; D=depth of the root zone; Sa=total available water height per unit soil depth at saturation; p=easily usable fraction.

Hypotheses considered during the construction of the agricultural model imply that it is validity is limited to the following situations:

$$0 \le ASI_i \le 1 \tag{8}$$

$$0.5ETm_i \le ETa_i \le ETm_i \tag{9}$$

The constraint related with the hydroelectric production tariff is:

$$P_{e\,i} = P_{e\max} \cdot \left[a \cdot \left(\frac{\eta_i \cdot \gamma \cdot QT_i \cdot H_i}{PINST \cdot NHP_i} \right)^2 + b \cdot \left(\frac{\eta_i \cdot \gamma \cdot QT_i \cdot H_i}{PINST \cdot NHP_i} \right) + c \right] \tag{10}$$

a, b, c = constants of the expression that gives the hydroelectric production tariff during period i; P_{emax}=maximum hydroelectric production tariff; *PINST*=installed capacity; NHP_i=number of hours of period i.

The mass balance equation in the reservoir is:

$$V_{i+1}=V_i+QA_i\text{-}QIR_i\text{-}QT_i\text{-}QD_i \tag{11}$$

V_i=water volume in the reservoir during period i; QA_i=inflow volume during period i; QIR_i=volume for the agricultural plant during period i; QT_i=volume used by the turbines during period i; QD_i=downstream release from the dam to the river bed by-passed by the hydraulic circuit during period i.

The elevation-storage curve at the reservoir:

$$V_i = d(CSA_i - \mathrm{CFA})^e \tag{12}$$

d, e = constants of the elevation-storage curve; CSA_i=reservoir water surface level during period i; *CFA*=reservoir bottom level.

The elevation-flow curve at the end of the hydraulic circuit:

$$CSR_i = CFR + \left[(QT_i + QD_i)/(3600 \cdot f \cdot NHP_i)\right]^{1/g} \tag{13}$$

CSR_i=water level at the end of the hydraulic circuit during period i; *CFR*=bottom level at the end of the hydraulic circuit during period i; *f, g* = constants of the elevation-flow curve.

The gross head is:

$$H_i = CSA_i - CSR_i \tag{14}$$

The reservoir water levels during each period are limited by:

$$CSA_i \geq CSAMIN_i \tag{15}$$

$$CSA_i \leq CSAMAX_i \tag{16}$$

$CSAMIN_i$=minimum allowable reservoir water level during period i; $CSAMAX_i$=maximum allowable reservoir water level during period i.

Downstream releases must satisfy:

$$QD_i \geq QCM_i \tag{17}$$

QCM_i = minimum downstream release from the dam to the river bed by-passed by the hydraulic circuit during period i;

$$QD_i + QT_i \geq QJMN_i \tag{18}$$

$QJMN_i$ = minimum admissible downstream release during period i.

$$QD_i + QT_i \leq QCR_i \tag{19}$$

QCR_i = maximum admissible downstream release not to be exceeded during period i;

The energy produced, during period i, is limited by the installed capacity:

$$\eta_i \cdot \gamma \cdot QT_i \cdot H_i \leq PINST \cdot NHP_i \tag{20}$$

The energy produced, during period i, must satisfy minimum demanded production:

$$\eta_i \cdot \gamma \cdot QT_i \cdot H_i \geq EMN_i \tag{21}$$

EMN_i = minimum demanded hydroelectric production during period i;

Stored water in the reservoir, in the beginning and in the end of the six month period, must be imposed by the user. If we want a stationary situation:

$$V_1 = V_N \tag{22}$$

The mathematical configuration of the decision model presented in this paper indicates that nonlinear programming is the appropriate method to use, this being done by applying the GAMS/MINOS solver (Brooke et al. 1998, Murtagh and Saunders 1995).

3. APPLICATION EXAMPLES

In order to check the computational feasibility and dynamic behavior of the decision model, several tests were conducted. Real data was changed or mixed with artificial data to create extreme fictitious situations that provided expressive illustrative examples. The agricultural land has an area of *A*=600 ha, a maximum agricultural production per hectare of *Ym*=6 t/ha and a unit price for the agricultural production of *Py*=160€/t. The agricultural land will be irrigated from the multipurpose reservoir during the 6 month crop period from March to August. Efficiency of the irrigation system is 70%. Other data associated with the agricultural production function, shown in Figure 1, can be found in Cunha et al. (1993).

In Table 1 we present some data associated with the multipurpose reservoir. The installed capacity is $PINST$=10 MW and the maximum hydroelectric production tariff is P_{emax}=8 cts/kwh. Other data can be found in Almeida and Cunha (1999).

Table 1: Data associated with the multipurpose reservoir

I	***QA_i (m^3)***	***QCM_i (m^3)***	***$QJMN_i$ (m^3)***	***QCR_i (m^3)***	***EMN_i (10^3kwh)***	***$CSAMIN_i$ (m)***	***$CSAMAX_i$ (m)***
1	2443890	(10000)	(100000)	(1000000)	100	517 (517)	536 (531)
2	2118140	(10000)	(100000)	(1000000)	100	517 (517)	536 (531)
3	1624010	(10000)	(70000)	(1000000)	100	517 (517)	536 (531)
4	1513950	(10000)	(70000)	(1000000)	100	517 (517)	536 (531)
5	1234500	(10000)	(50000)	(1000000)	25	517 (517)	536 (532)
6	671630	(10000)	(50000)	(1000000)	25	517 (517)	536 (532)
7	259130	(10000)	(20000)	(500000)	25	517 (520)	536 (536)
8	76570	(10000)	(20000)	(500000)	25	517 (520)	536 (536)
9	122200	(10000)	(20000)	(500000)	25	517 (520)	536 (536)
10	39550	(10000)	(20000)	(500000)	25	517 (520)	536 (536)
11	28390	(10000)	(20000)	(500000)	25	517 (520)	536 (536)
12	30830	(10000)	(20000)	(500000)	25	517 (520)	536 (536)

3.1 EXAMPLE 1

In this first example we will consider that the multipurpose reservoir has no hydroelectric facilities. Furthermore we will consider that only the constraints expressed in the last two columns of Table 1, with values that are not between parentheses, are active. Once the multipurpose operation is quite free from constraints on water use, the model is able to implement an irrigation policy leading to maximum production. Optimum irrigation during each bimonthly vegetative period, next to the plants, is I_1=615 mm, I_2=314 mm, I_3=151 mm (which corresponds to the upstream diversion from the reservoir of $QIR_1=4*1317160$ m^3, $QIR_2=4*672080$ m^3 and $QIR_3=4*322950$ m^3 respectively). The maximum agricultural production is achieved, Ya/Ym=1, and the corresponding maximum agricultural remuneration is €576 000. Figure 4 shows the allocation of water in the 12 fortnight periods.

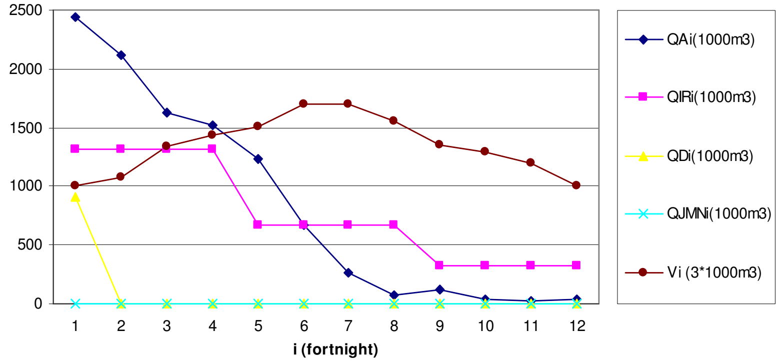

Figure 4: Optimum water allocation in the 12 fortnight periods for example 1

3.2 EXAMPLE 2

In this second example we will still consider that the multipurpose reservoir has no hydroelectric facilities. However, all the constraints of Table 1 associated with water use are now activated. The values to be considered are those between parentheses.

In the first fortnight periods we imposed lower maximum admissible pool levels to represent flood control constrains. In the last fortnight periods we imposed higher minimum admissible pool levels to represent reserved water supply volume constraints.

Due to the activation of these constraints it will be no longer possible to achieve the maximum agricultural production. Here the optimum agricultural production is reduced to *Ya/Ym*=0.61, which corresponds to an optimum agricultural remuneration of €354 124. Irrigation during each bimonthly vegetative period, next to the plants, is I_1=615 mm, I_2=142 mm, I_3=120 mm (which corresponds do the upstream diversion from the reservoir of $QIR_1=4*1317160$ m^3, $QIR_2=4*303410$ m^3 and $QIR_3=4*257700$ m^3 respectively). From Figures 5 and 6 we can clearly see that the model uses all the storage capacity compatible with the constraint on $CSAMAX_6$ in order to irrigate the agricultural plant as much as possible in the two last vegetative periods.

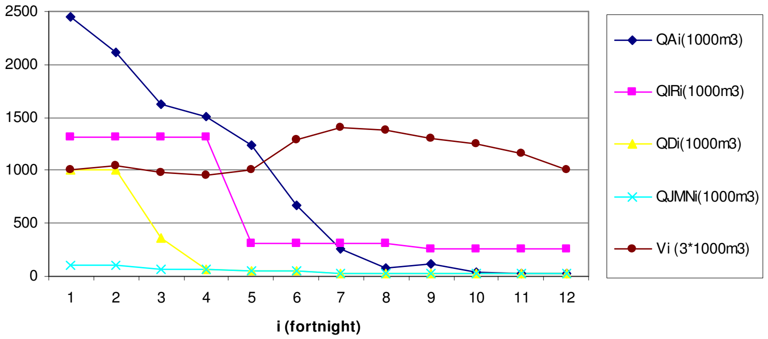

Figure 5: Optimum water allocation in the 12 fortnight periods for example 2

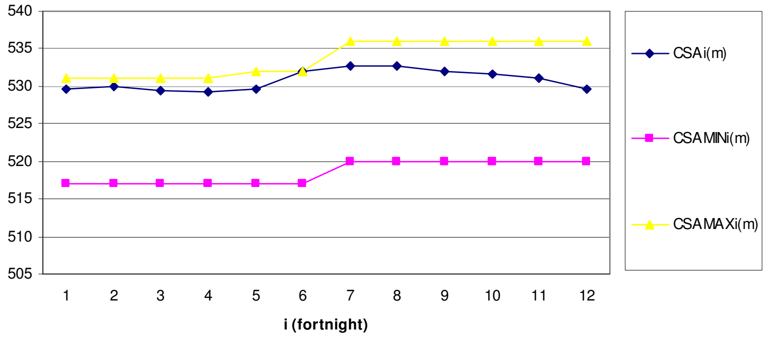

Figure 6: Surface water level in the 12 fortnight periods for example 2

3.3 EXAMPLE 3

In this third example we will go back to the conditions of example 1. However we will now introduce the hydroelectric production. Our aim is to test how the model responds, in this situation free from constrains on water use, to the possibility of allocating water to hydroelectric production. The hydroelectric production tariff price is given by equation (10).

In this case the optimum solution will not necessarily lead to integral satisfaction of agricultural irrigation demands, because hydropower production is a competing alternative use. Here, the optimum solution corresponds to an agricultural production reduced to Ya/Ym=0.74, resulting in an optimum agricultural remuneration of €424 143. Irrigation during each bimonthly vegetative period, next to the plants, is I_1= 236 mm, I_2= 392 mm, I_3=57 mm (which corresponds do the upstream diversion from the reservoir of $QIR_1=4*505500\ \text{m}^3$, $QIR_2=4*840900\ \text{m}^3$ and $QIR_3=4*121900\ \text{m}^3$ respectively).

The total volume allocated to hydroelectric production is $\sum_{i=1}^{12} QT_i = 4289600\ \text{m}^3$, which gives a hydroelectric production remuneration of €224 344.

Global remuneration reaches €648 487, which is more than the agricultural remuneration of example 1, as one would expect once there was a water deficit.

Figure 7 shows the water volume allocation for the 12 fortnight periods.

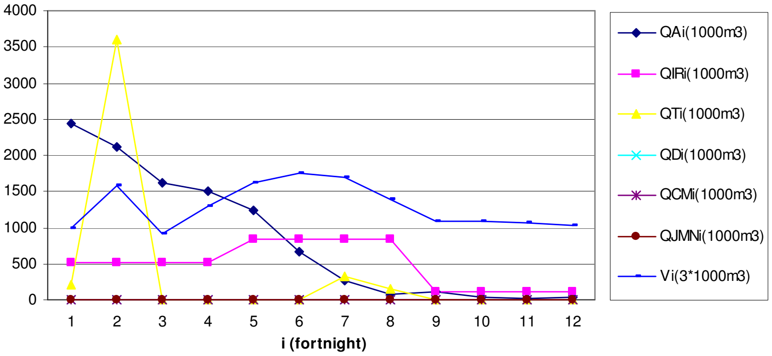

Figure 7: Optimum water allocation in the 12 fortnight periods for example 3

Finally we introduced all the constraints of example 2 and the minimum hydroelectric production constraints shown in column 6 of Table 1. As expected, the global remuneration fell to €373 220.

The executable model file measures 2Mb. The execution time is less than 1 minute with a Pentium 4 processor.

4. CONCLUSIONS

The approach presented here was able to provide the optimum allocation of water for agricultural production, for hydroelectric production and for downstream releases. From a planning point of view this approach can be used as a criterion for water sharing when competing, mutually exclusive, water uses are involved.

The decision model proved to be feasible from a computation point of view.

From a dynamic point of view the decision model showed a logical and consistent response to the modifications introduced in the data and in the constraints.

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