AN OPTIMIZATION HEURISTIC FOR THE HIGHWAY HORIZONTAL ALIGNMENT PROBLEM

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ABSTRACT

In this research, we present an optimization heuristic to solve the horizontal alignment of a highway segment. The iterative heuristic uses a two-tiered process to solve for a piecewise linear line that approximates the horizontal alignment. In each iteration, the outer tier uses a neighborhood search approach to find good orientations for each segment of the piecewise linear line, and the inner tier solves for the optimal lengths as well as the locations of each segment with a mixed integer program (MIP). Constraints in the MIP ensures that the piecewise linear line comes across each of the control areas in a given set. The optimal objective function value returned by the inner tier is used by the outer tier to compare the quality between different solutions. The lengths of each segment are also properly constrained to ensure that curves can be correctly deployed afterwards. Starting from an initial feasible solution, the process gradually improves the alignment through iterations. A computational example is provided.

KEY WORDS

highway alignment, automated design, integer programming, optimization, genetic algorithm.

INTRODUCTION

The task of highway geometric design is to determine the horizontal alignment, vertical alignment, and the cross section of a highway. Restrictions on the horizontal alignment mainly come from code requirements (for example, AASHTO 2001) and external limitations. Design codes require that the horizontal alignment of a highway be composed of three types of design elements, namely line segments which are part of a line, circular arcs which are part of a circle, and spirals whose curvature changes smoothly and continuously. The spiral is the most complicated element type among the three design elements. A number of functions can be used as spirals. In this heuristic, we employ the widely used clothoid curve. The family of clothoid curves can be described with the function $A^2 = RL$, where A is a parameter whose value determines the exact shape of the individual curve, R is the curvature radius of the curve at a point p on the curve, and L is the distance from the starting point of the curve to point p, measured along the curve. It is easy to see that the radius is infinity at the starting point, and decreases smoothly along the curve. Properties of the clothoid curve

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can be found elsewhere (AASHTO 2001 and Wang et al. 2001). The code provides detailed and strict requirements on how these design elements can be deployed. In short, the ending point of one element has to match the starting point of the next element in position and azimuth. In addition, the radius has to change continuously and smoothly along the highway. Since both the line segments and circular curves have a fixed radius, it follows that at least one spiral element has to be inserted between any two non-spiral elements. The exact shape of each design element is defined by a set of parameters: the parameter A of spirals, the radius of circular curves, and others. The code imposes further restrictions on the allowed ranges of these parameters, as well as how these parameters should be coordinated among each other.

The external limitations considered in this research are the control areas. A control area is a region where the highway should cross. Here, we assume that all control areas are rectangular in shape. Areas that are significantly non-rectangular can be approximated by a combination of multiple rectangles.

Due to the nature of the design elements and strict requirements by design codes, the design work is complicated and time consuming. In practice, the geometric design of a highway project is done manually by experienced engineers. Although the horizontal and vertical alignments are deeply related, and coordination between horizontal and vertical alignments are important (Smith and Lamm, 1994 and Hassan et al., 1997), engineers typically design the horizontal alignment first, then develop the vertical alignment according to the horizontal alignment. The mathematics involved in the design of horizontal alignment is well known (Easa et al. 2002), but the computing work is complicated. Human engineers can produce remarkably good designs. However, under tight time and budget constraints, the resulting design is often chosen from a handful of candidates, as opposed to being generated with a systematic optimization process. Motivated by the need of an automated and optimization oriented highway alignment model, researchers have been developing highway alignment-related models for at least 30 years. In particular, Tuner and Miles (1971), Tuner (1978), Parker (1977), Jong (1998), and Jha (2003) developed models that attempt to optimize the horizontal alignment of a given highway section. All these research results are far from what engineers use in practice because design elements are not fully considered in the models. In this paper, we propose an optimization-oriented heuristic that enables the computer to solve for a highway horizontal alignment, which takes the three design element types into consideration. The solution process is an efficient genetic algorithm heuristic, which uses a linear program as its inner core. The heuristic can be solved very quickly and it yields good solutions.

The structure of this paper is as follows. Following this introduction, we will introduce the heuristic as well as the solution process in the next section. The third section presents a computational example. The final section concludes this paper and lists some promising directions for future research.

THE HEURISTIC

OVERVIEW

The heuristic developed in this study approximates the highway alignment with a piecewise linear line. A piecewise linear line is essentially a set of line segments connected together consecutively. The connecting points are referred to as IP points. Figure 1 illustrates a piecewise linear line that approximates a horizontal alignment, and design elements that define the actual alignment.

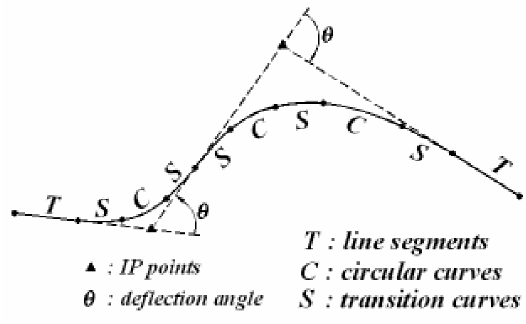

Figure 1. Illustration of a piecewise linear line and corresponding design elements.

A piecewise linear line can be viewed as an ordered set of line segments, which is characterized by the number of line segments it contains, the positions of the connecting points, and the azimuth of each line segment. The heuristic solves for an optimized piecewise linear line in two tiers. The outer tier is a genetic algorithm heuristic that generates partial solutions, which only includes the number of lines as well as their azimuths. Each partial solution is then passed on to the inner tier, which uses a linear mixed integer program (MIP) to complete the piecewise linear line. This piecewise linear line is also referred to as a solution. The MIP solves for the lengths of the line segments and this effectively determines the positions of each IP point. The optimal objective function value of the MIP, which quantitatively represents the quality of that alignment, is then passed back to the outer tier to determine the survival of the corresponding solutions in subsequent iterations. The final output is the best solution encountered in the solving process. When the outer tier generates the azimuths for the lines, a screening process ensures that the deflection angles at all IP points satisfy code requirements. In the inner tier, a set of constraints ensure that the length of each line segment is sufficient for the appropriate deployment of the design elements. Therefore, the final piecewise linear line allows the actual design elements be deployed easily to complete the highway alignment. Next, we introduce the two tiers in detail, starting from the inner tier.

INNER TIER

The inner tier takes as given an ordered set of lines whose azimuths are set, and uses a MIP model to assemble the lines into a piecewise linear line by solving for the length of each line segment. These lengths will uniquely determine the piecewise linear line. The design elements themselves are not directly included in the MIP model, but the parameters of the

MIP are constructed in a way that the actual design elements complying with code requirements can later be deployed easily along the resulting piecewise linear line. Parameters and decision variables used in the MIP are defined as follows. N is the number of line segments, and $L = \{0,1,\ldots,N-1\}$ is the ordered set of all line segments. The corresponding IP points are numbered from 0 to N, where point 0 is the starting point of the highway. CA = {0,1,...,C-1} is the set of control areas. The starting point (X_s,Y_s) and ending point (X_t,Y_t) of the highway are also given. The coordinate of IP point i is (x_i,y_i), where x_i and y_i are decision variables in the model. Another decision variable l_i represents the length of line segment i. For each line segment i, we also define a binary variable k_i, which takes the value 0 if l_i = 0, and takes the value 1 if $l_i \geq 0$. The binary variable p_{ij} = 0 if line segment i intersects control area j, and if p_{ij} = 1 the model does not restrict the relative location between line segment i and control area j. For each line segment i the azimuth θ_i is generated in the outer tier, thus it is regarded as a given constant in the inner tier. By simple geometry the length of line segment i is a linear function of the x-coordinates of its two ends, which can be expressed as $l_i = (x_{i+1} - x_i)/\cos\theta_i$.

Under the condition that the number of line segments as well as the azimuths of all of them are given and fixed, the following MIP model minimizes the total length of the piecewise linear line, subject to certain constraints. Every feasible solution of the model contains a set of lengths for each of the line segments so that inserting design elements is possible for the resulting piecewise linear line. If a feasible solution exists, the optimal solution of the model will yield the feasible solution (which is a piecewise linear line) that has the minimum total length. Note that the length of the piecewise linear line is always greater than that of the highway after all design elements are inserted, but the difference should not compromise the ability of the outer tier to distinguish the better solutions from the rest. The model is listed below.

The objective function (1) minimizes the total length of the piecewise linear line.

Minimize $z = \sum_{i \in L} l_i$ (1)

The constraints (2) to (5) defines the starting point and ending point of the piecewise linear line, where (X_s, Y_s) and (X_t, Y_t) are the coordinates of the given starting and ending points, respectively.

$$x_0 = X_s \quad (2)$$

$$y_0 = Y_s \quad (3)$$

$$x_N = X_t \quad (4)$$

$$y_N = Y_t \quad (5)$$

Constraints (6) and (7) establish the relationship between the locations of the IP points and the lengths of the line segments.

$$l_i = \frac{x_{i+1} - x_i}{\cos\theta_i} \quad \forall i \in L \tag{6}$$

$$l_i = \frac{y_{i+1} - y_i}{\sin\theta_i} \quad \forall i \in L \tag{7}$$

The nature of this MIP model requires that the number of line segments that make up the alignment should be given. To comply with this requirement, and at the same time have the flexibility to adjust the number of line segments within this model, we fix the number of line segments, but let the model collapse some of them by setting their lengths to 0. Doing so effectively enables the model to eliminate some of the given line segments from the resulting alignment. A complicating factor is that the deflection angle at the end of line segment *i* is $\theta_{i+1} - \theta_i$ or $\theta_{i+2} - \theta_i$, depending on if line segment *i*+1 is crushed. This deflection angle in turn affects the minimum required lengths of line segment *i*, *i*+1, or *i*+2, depending on the status of line segment *i*. To simplify the problem, we exclude the possibility of collapsing any two consecutive line segments by including constraint (8). The effect of collapsing line segments on deflection angles are discussed next.

$$k_{i-1} + k_i + k_{i+1} \geq 2 \quad \forall i \in L \setminus \{0,1,N-2,N-1\} \tag{8}$$

The minimum length of line segment *i* depends on the deflection angles at its two ends, this in turn depends on whether line segments *i*-1, *i* or *i*+1 are crushed. The constants $L_i^{\min 1}$, $L_i^{\min 2}$, and $L_i^{\min 3}$ are the minimum length of line segment *i* when none of the three line segments are collapsed, when line segment *i*-1 is collapsed, and when line segment *i*+1 is collapsed, respectively (when line segment *i* itself is collapsed, its length is 0). These constants are computed in such a way that the distance between neighboring IP points is sufficient for the deployment of appropriate design elements according to the deflection angles at the IP points. Detailed computation process for this minimum length is lengthy, and is omitted from this paper due to page limits. The design code does not specify the maximum distance between neighboring IP points. Upper limits $L_i^{\max 1}$, $L_i^{\max 2}$, and $L_i^{\max 3}$ can be set to an appropriate number to prevent driving fatigue due to long straight highways, or to a large number if there is no such need. The following pair of constraints establishes the correct limits on the line segment lengths according to the way the line segments are collapsed. If line segment *i* is collapsed, $k_i=0$, $k_{i-1}=k_{i+1}=1$, and these constraints will ensure that $l_i=0$.

$$l_i \geq (k_{i-1} + k_i + k_{i+1} - 2) \times L_i^{\min 1} + (1 - k_{i-1}) \times L_i^{\min 2} + (1 - k_{i+1}) \times L_i^{\min 3} \; \forall i \in L \setminus \{0,1,N-2,N-1\} \tag{9}$$

$$l_i \leq (k_{i-1} + k_i + k_{i+1} - 2) \times L_i^{\max 1} + (1 - k_{i-1}) \times L_i^{\max 2} + (1 - k_{i+1}) \times L_i^{\max 3} \; \forall i \in L \setminus \{0,1,N-2,N-1\} \tag{10}$$

Because the highway under design should smoothly connect to existing highways, the first and last line segments are never crushed to fix the azimuths at the two ends, which are assured by constraint (11), and their lengths are bounded by the constraints (12) and (13).

$$k_0 = k_{N-1} = 1 \tag{11}$$

$$L_0^{\min} \le l_0 \le L_0^{\max} \quad (12)$$

$$L_{N-1}^{\min} \le l_{N-1} \le L_{N-1}^{\max} \quad (13)$$

The next set of constraints defines the condition that a specified line segment i comes across a given control area j. Recall that the angles θ_i are input parameters to the MIP model. Figure 2 shows the case where the angle θ_i falls in the first quadrant. Referring to this figure, the control area is the rectangle in solid line, and line segment i itself is shown in thick line with its two ends marked. If the point (x_i,y_i) falls in the hexagon defined by the points A, B, C, $\left(X_j^U, Y_j^L\right)$, $\left(X_j^U, Y_j^U\right)$, and $\left(X_j^L, Y_j^U\right)$, then line segment i will come across the control area. Constraints (14) to (19) are developed for this purpose, where M is a large positive constant. If p_{ij} = 0 these constraints together will force line segment i to come across control area j. If p_{ij} = 1 none of these constraints will become binding.

$$x_i + Mp_{ij} \ge X_j^L - l_i \times \cos\theta_i \qquad \forall i \in L,\, j \in CA \quad (14)$$

$$x_i - Mp_{ij} \le X_j^U \qquad \forall i \in L,\, j \in CA \quad (15)$$

$$y_i + Mp_{ij} \ge Y_j^L - l_i \times \sin\theta_i \qquad \forall i \in L,\, j \in CA \quad (16)$$

$$y_i - Mp_{ij} \le Y_j^U \qquad \forall i \in L,\, j \in CA \quad (17)$$

$$y_i - Mp_{ij} \le Y_j^U + \left(x_i - X_j^L\right)\tan\theta_i \qquad \forall i \in L,\, j \in CA \quad (18)$$

$$y_i + Mp_{ij} \ge Y_j^L + \left(x_i - X_j^U\right)\tan\theta_i \qquad \forall i \in L,\, j \in CA \quad (19)$$

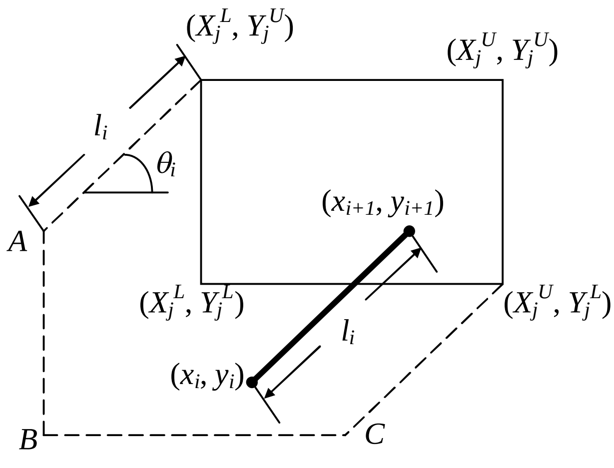

Figure 2. Illustration of a control area and a line segment that comes across from the first quadrant.

Constraints corresponding to the other cases where the angle θ_i falls in the second, third, or fourth quadrant are similar. They are listed below without detailed explanation. For the second quadrant, we need constraints (16), (17), and (20) to (23). For the third quadrant, we need constraints (18) to (21), and (24), (25). For the fourth quadrant, we need constraints (14),(15),(22), and (24) to (26).

$$x_i + Mp_{ij} \geq X_j^L \qquad \forall i \in L,\ j \in CA \tag{20}$$

$$x_i - Mp_{ij} \leq X_j^U - l_i \times \cos\theta_i \qquad \forall i \in L,\ j \in CA \tag{21}$$

$$y_i - Mp_{ij} \leq Y_j^U + \left(x_i - X_j^U\right)\tan\theta_i \qquad \forall i \in L,\ j \in CA \tag{22}$$

$$y_i + Mp_{ij} \geq Y_j^L + \left(x_i - X_j^L\right)\tan\theta_i \qquad \forall i \in L,\ j \in CA \tag{23}$$

$$y_i + Mp_{ij} \geq Y_j^L \qquad \forall i \in L,\ j \in CA \tag{24}$$

$$y_i - Mp_{ij} \leq Y_j^U - l_i \times \sin\theta_i \qquad \forall i \in L,\ j \in CA \tag{25}$$

$$y_i + Mp_{ij} \geq Y_j^L - \left(x_i - X_j^L\right)\tan\theta_i \qquad \forall i \in L,\ j \in CA \tag{26}$$

The following constraint (27) ensures that at least one line segment has to come across each control area. Note that the order the highway passes each control area is left for the model to determine optimally.

$$\sum_{i=0}^{N-1} p_{ij} \leq N - 1 \qquad \forall j \in CA \tag{27}$$

Constraint (27) imposes the requirement in the “hard” sense, in that the resulting piecewise linear line will not be regarded as feasible unless it passes each of the control areas. Alternatively, this same requirement can also be expressed in a soft sense as constraint (28), where E_j is either 0 or 1 as stated in constraint (30). The variables E_j is also added to the objective function (1) as a penalty term with a large positive coefficient α as shown in (29). In this way the piecewise linear line is allowed to skip some or all of the control areas and still remain feasible. When this happens, a penalty proportional to the number of skipped control areas will be applied to the objective function value. The advantage of this soft form is the model can still obtain an optimal solution even when it is impossible for the combination of lines to come across all control areas and satisfy all other constraints at the same time. At early stages of the solution process when solutions that can satisfy the hard form is scarce, objective function values yielded by the soft form is important to guide the outer tier towards better solutions.

$$\sum_{i=0}^{N-1} p_{ij} \leq N - 1 + E_j \qquad \forall j \in CA \tag{28}$$

$$\text{Minimize } z = \sum_{i \in L} l_i + \alpha \sum_{j \in CA} E_j \tag{29}$$

$$E_j \in \{0,1\} \qquad \forall j \in CA \tag{30}$$

Finally we have non-negativity constraints and binary integer constraints (31) and (32).

$$l_i \geq 0 \quad \forall i \in L \tag{31}$$

$$k_i \in \{0,1\} \quad \forall i \in L \tag{32}$$

OUTER TIER

The outer tier uses a genetic algorithm process to search for an appropriate number of line segments as well as a good combination of their azimuths. The azimuth of a line segment is a continuous real number between 0 and 2π. In the coding for the genetic algorithm, we represent the azimuth with 9 bits, and a genome is composed of all the bits of the azimuths of all its line segments. Standard cross-over and mutations are used to search for new solutions.

Each solution generated in the genetic algorithm consists of an ordered set of lines whose azimuths are known. This information is used by the inner tier to establish the MIP model. If the model is feasible, its optimum objective function value is returned back to the outer tier. If the model is infeasible, a large number is returned. The genetic algorithm routine uses this returned value to compute the fitness value of the solution, which in turn determines the survival probability of this solution in subsequent iterations. A flow chart of the solution process is shown in Figure 3.

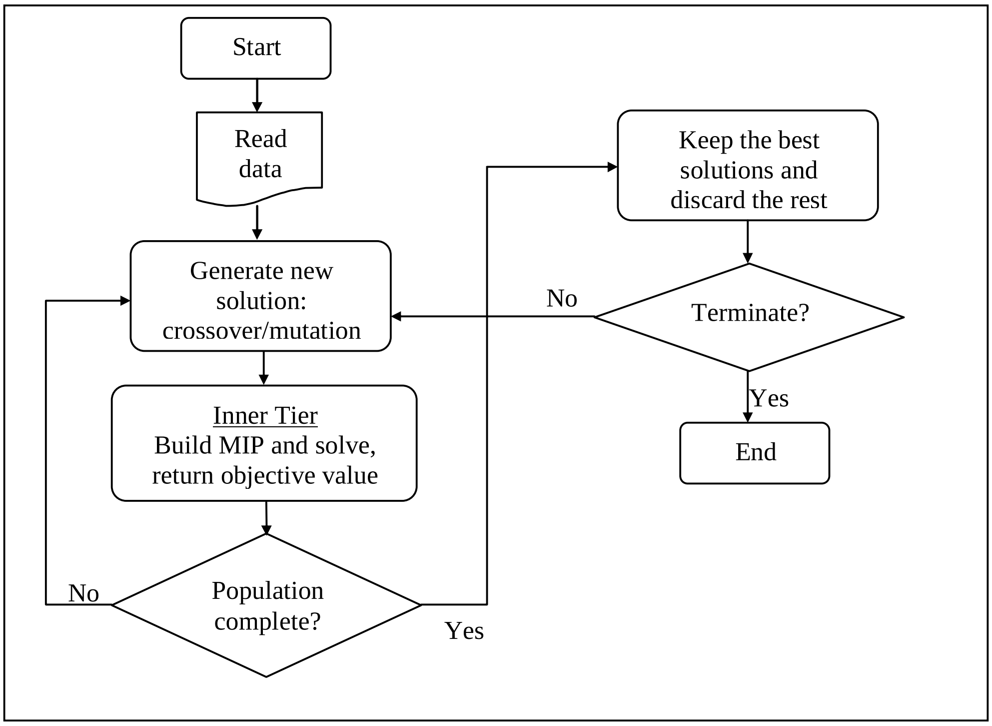

Figure 3. Flow chart of the solution process.

NUMERICAL EXAMPLE

One numerical example is presented in this section. This example demonstrates the correctness and efficiency of the heuristic, as well as the ability to adjust the resulting alignment according to different requirements. The highway starts at (0,450) with azimuth set to 0.5 radian, and ends at (1500,500) with azimuth equal to 5.8 radian. There are two control areas which the highway is required to come across. Figure 4 shows the final piecewise linear line. The total length of the piecewise linear line is 1766 meters, and the computation took less than 10 seconds.

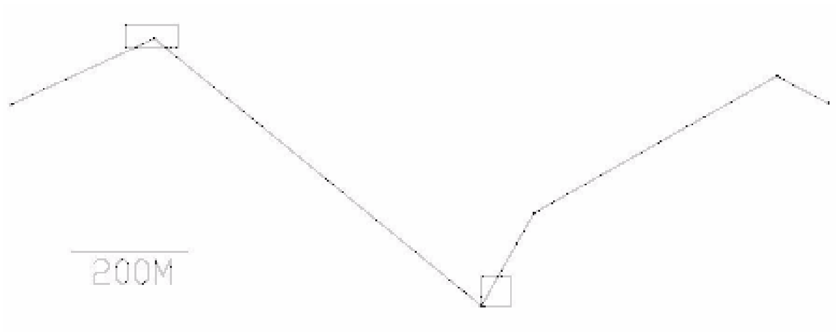

Figure 4. Solution of the example.

CONCLUSION

Geometric design is a fundamental part of any highway design project, and designing the horizontal alignment is one of the most complicated parts in geometric design. In this research, we presented an optimization oriented heuristic that can yield a piecewise linear line that approximates a highway horizontal alignment. The heuristic is composed of two tiers. The outer tier generates partial solutions by searching for good combinations of the number of lines as well as their azimuths. The inner tier uses a MIP to complete the solution by determining the lengths of each line segment. A genetic algorithm process guides the outer tier to generate solutions, weed out the weaker, and keep the better ones. The heuristic is designed in a way that the circular curves, line segments, and clothoid curves can be deployed along the resulting piecewise linear line fulfilling code requirements. How to incorporate vertical alignment considerations into this heuristic is an important direction for future research.

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