OPTIMAL SITING OF SOLID WASTE TRANSFER STATIONS FOR MINIMIZING HAUL COSTS

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ABSTRACT

Large solid waste management systems use transfer and transoprt operations to haul wastes in bulk quantities to remotely located disposal facilities. Proper selection of the location for a transfer operation with respect to the disposal sites is an important consideration for minimizing system operating costs. When several transfer locations are needed to supply wastes to meet the capacities of multiple disposal sites, cost comparisons between alternate locations will allow the optimal siting of a transfer station to minimize haul costs. The present paper illustrates the use of mathematical analysis involving linear programming as an effective tool for the selection of transfer station locations. Solutions for alternate scenarios are easily accomplished by the use of a computer program.

KEY WORDS

solid waste disposal, siting of transfer stations. linear programming, computer applications.

INTRODUCTION

Solid wastes collected from various residential and commercial sites are hauled using small capacity collection vehicles. When the waste disposal sites are located relatively far from the collection routes, direct hauling becomes economically infeasible. Solid waste management systems handling large volumes of waste use transfer and transport operations as part of the system to achieve cost savings. In such systems the collection vehicles deliver the wastes to one or more transfer stations. High capacity transport trailers are then used to haul the wastes to disposal facilities sited at remote locations.

The cost of transport operations can be established based on available travel routes between the transfer stations and the disposal sites. A major factor in achieving cost savings in the system operation is the location of the transfer station with respect to the disposal sites. To identify the optimal site for a transfer station that will provide the minimum operating costs it will be necessary to compare the transport costs between alternate transfer station locations.

The problem of selecting the transfer station location for most economical system operation becomes more complex when the system operates with several transfer stations and several disposal sites. Application of linear programming methods through the use of computer codes can provide fast solutions for such complex situations.

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MATHEMATICAL MODEL

A cost function, CF relating the total costs for transporting wastes from [n] number of transfer stations to [m] number of disposal sites can be represented by

$$CF = \sum_{j=1}^{m} \sum_{i=1}^{n} X_{ij} C_{ij}$$

where X_{ij} = amount of waste hauled from transfer station i to disposal site j
C_{ij} = cost of hauling waste from transfer station i to disposal site j

The round-trip transport cost for hauling wastes, C_{ij} is usually determined by multiplying a unit cost in dollars per hour with the haul time in hours. The haul time is represented by a relation of the form (a + b x) where a and b are haul-time constants in hours per trip, and hours per mile, respectively and x is round-trip haul distance in miles.

$$C_{ij} = (\$/hr)(a + b\, x_{ij})$$

where x_{ij} = round-trip distance in miles between transfer station i and disposal site j.

The following constraints apply to the problem:

(1) total amount of wastes hauled to all disposal sites must equal the amount of wastes delivered to the transfer station

$$\sum_{j=1}^{m} X_{ij} = R_i \quad \text{for } i = 1 \text{ to } n$$

where R_i = amount of waste delivered to transfer station i

(2) each disposal site can accept only a specified amount of wastes

$$\sum_{i=1}^{n} X_{ij} \le D_j \quad \text{for } j = 1 \text{ to } m$$

where D_j = amount of waste that can be accepted at disposal site j

(3) wastes hauled from each transfer station is equal to or less than zero (it cannot have negative values)

$$X_{ij} \ge 0$$

An optimum solution to the problem is obtained by minimizing the cost function satisfying all the constraints. This can be done with linear programming methods using the simplex algorithm. However, since the simplex algorithm is usually encoded to maximize functions, it is necessary to convert the problem from a minimization to a maximization problem using the Dual Method. The matrix iterations associated with the simplex method can be easily performed by developing a computer code. An algorithm for developing the computer code is presented in Figure 1.

The following numerical example illustrates the application of the linear programming method for the optimum site selection for a solid waste transfer station.

NUMERICAL EXAMPLE

A solid waste management system operating with four disposal sites (D1, D2, D3, D4) and three transfer stations (T1, T2, T3) is considering the addition of a fourth transfer station to handle additional waste quantities. The problem is to determine which of two possible sites (T4 and T5) will provide cost effective operation.

The capacities of the four disposal sites are 4, 10, 3 and 8 units per day while the wastes delivered to the three existing transfer stations are 3, 3, and 5 units/day. The new transfer station to be added will receive additional waste amounts of 2 units/day. Based on street layout the round-trip travel distances between transfer stations and disposal sites, in miles, are given by Table 1.

Table 1. Round-Trip Travel Distances in Miles for Numerical Example

Round-Trip Haul Distance (Miles)					
	T1	**T2**	**T3**	**T4**	**T5**
D1	30	24	40	30	20
D2	20	26	10	20	38
D3	48	30	30	8	26
D4	24	42	42	64	54

The haul costs for transporting the wastes are estimated to be at $35 per hour and the transport time in hour per trip is given by [0.08 hour/trip + 0.025 hour per mile]. Based on this, the actual haul costs per trip, in dollars, can be calculated as in Table 2.

Table 2. Haul Costs in Dollars per Trip for Numerical Example

Haul Costs (Dollars per Trip)					
	T1	**T2**	**T3**	**T4**	**T5**
D1	29.05	23.80	37.80	29.05	20.30
D2	20.30	25.55	11.55	20.30	36.05
D3	44.80	29.05	29.05	9.80	25.55
D4	23.80	39.55	39.55	58.80	50.05

```
Input n,m, A(n+1,m): For I =1,….,n → Input A_{i,2m+1}: rof
n1:= n+1: m1:= m+1: mm1:= m+n+1
for I = 1,….,n →
    for j = m+1,….,mm1−1 →
        if j = m+1 → A_{i,j} := 1 else A_{i,j} := 0: fi
        rof
rof  { end creation of initial feasible solution}
t := 0: t1 :=1
do until t1 =0
    for I = 1,…,mm1 −1 →
        if A_{n1,I} < 0 →
            if  A_{n1,I} <  t → t := A_{n1,I}: p := i: fi
        fi
        rof : if t ≥ 0 → t1:= 0: fi
if p > 0 → u := 1e20: s := 1
    for I = 1,…,n  →
        if A_{i,p} > 0 →
            w:= A_{i,mm1}/A_{i,p}
            if u ≥ w →
              if t = w and s ≤ 1 → q := I: s := s+1: fi
                u := w: r := i.                fi
        fi
    rof
    if s > 1 → d := 1  {degeneracy exists}
        for j = 1,…,mm1 −1
            if A_{r,j}/A_{r,p} > A_{q,j}/A_{q,p} and d ≤ 1 →
               r := q: p := j: d := d+1
            fi
        rof
    fi  {end handling degeneracy}
    if A_{r,p} ≠ 1 and A_{r,p} > 0 → z := A_{r,p}
      for j = 1,…,mm1  → A_{r,j} := A_{r,j}/z:  rof
    fi
    for I = 1,…,n1
        if i≠ r → g:= A_{i,p}
          for j = 1,…,mm1
              A_{i,j}:= A_{i,j} − g A_{r,j}
          rof
        fi
   rof
```

```
fi: bd := 1:  j := 1:  ss :=0
        do until j ≥ mm1−1 →  {check for unboundedness}
            if A_{n1,j} < 0 →
              for I = 1,…,n  →
                  if A_{i,j} < 0 → ss:= ss +1:  fi
              rof
            fi
            if ss := n → j := mm1−1:  bd := 0:  t1 := 0  else ss := 0 fi: j = j+1
        od  {end of check for unboundedness}
od: f = 1
if bd = 1 → j := m+1: ss := 0  {check for infeasible solution}
       do until j ≥ mm1−1
           for I = 1,…,n  →
               if A_{i,j} = 0 → ss := ss+1  elseif A_{i,j} = 1 → k := 1: fi
           rof
           if ss := n−1 and k := 1 → j := mm1−1: f := 0 else ss := 0: fi:  j := j+1
       od
fi  {end check for infeasible solution}
if f = 0 →
  output message: No feasible solution is found
else output message: Problem is bounded
fi
ck := 0
    for j = 1,…,m  →
         for I = 1,…,n
              if A_{i,j} = 0 →
                   ck := ck +1
              elseif  A_{i,j} = 1
                         y :=  A_{i,j}:  r := I:  p := j
              fi
          rof
          if ck = n −1 and y = 1 →
             x_p := A_{r,mm1}:  output  x_p
          else → x_j := 0: output  x_j
          fi
             y := 0:  r := 0:  p := 0:  ck := 0
    rof: p
output A_{n1,mm1} as optimum value.    [End of Algorithm]
```

Figure 1. Algorithm for Simplex Method of Linear Programming

To determine the proper selection of transfer station between T4 and T5, the problem will have to be solved twice.

Linear Programming Model for Scenario 1: Locate Transfer Station at T4

The cost function to be minimized is represented by

$$CF = 29.05\,X_{11} + 20.30\,X_{12} + 44.80\,X_{13} + 23.80\,X_{14} + 23.80\,X_{21} + 25.55\,X_{22} + 29.05\,X_{23} + 39.55\,X_{24} + 37.80\,X_{31} + 11.55\,X_{32} + 29.05\,X_{33} + 39.55\,X_{34} + 29.05\,X_{41} + 20.30\,X_{42} + 9.80\,X_{43} + + 58.80\,X_{44}$$

subject to the following constraints:

$$X_{11} + X_{12} + X_{13} + X_{14} = 3 \qquad X_{11} + X_{21} + X_{31} + X_{41} \leq 4$$
$$X_{21} + X_{22} + X_{23} + X_{24} = 3 \qquad X_{12} + X_{22} + X_{32} + X_{42} \leq 10$$
$$X_{31} + X_{32} + X_{33} + X_{34} = 5 \qquad X_{13} + X_{23} + X_{33} + X_{43} \leq 3$$
$$X_{41} + X_{42} + X_{43} + X_{44} = 2 \qquad X_{14} + X_{24} + X_{34} + X_{44} \leq 8$$
$$X_{ij} \geq 0$$

Since the cost function needs to be minimized, the constraint equations are first converted to the following Primal equations:

$$- X_{11} - X_{12} - X_{13} - X_{14} \geq - 3 \qquad -X_{41} - X_{42} - X_{43} - X_{44} \geq -2$$
$$X_{11} + X_{12} + X_{13} + X_{14} \geq 3 \qquad X_{41} + X_{42} + X_{43} + X_{44} = 2$$
$$-X_{21} - X_{22} - X_{23} - X_{24} \geq -3 \qquad - X_{11} - X_{21} - X_{31} - X_{41} \geq 4$$
$$X_{21} + X_{22} + X_{23} + X_{24} \geq 3 \qquad - X_{12} - X_{22} - X_{32} - X_{42} \geq 10$$
$$-X_{31} - X_{32} - X_{33} - X_{34} \geq -5 \qquad - X_{13} - X_{23} - X_{33} - X_{43} \geq 3$$
$$X_{31} + X_{32} + X_{33} + X_{34} \geq 5 \qquad - X_{14} - X_{24} - X_{34} - X_{44} \geq 8$$
$$X_{ij} \geq 0$$

To convert the problem to a maximization one, the Dual equations become the following:

Maximize: $F = -3Z_1 + 3Z_2 - 3Z_3 + 3Z_4 - 5Z_5 + 5Z_6 - 2Z_7 + 2Z_8 - 4Z_9 - 10Z_{10} - 3Z_{11} - 8Z_{12}$

subject to:

$-Z_1 + Z_2 - Z_9 \leq 29.05$	$-Z_3 + Z_4 - Z_{11} \leq 29.05$	$-Z_7 + Z_8 - Z_9 \leq 29.05$
$-Z_1 + Z_2 - Z_{10} \leq 20.30$	$-Z_3 + Z_4 - Z_{12} \leq 39.55$	$-Z_7 + Z_8 - Z_{10} \leq 20.30$
$-Z_1 + Z_2 - Z_{11} \leq 44.80$	$-Z_5 + Z_6 - Z_9 \leq 37.80$	$-Z_7 + Z_8 - Z_{11} \leq 9.80$
$-Z_1 + Z_2 - Z_{12} \leq 20.30$	$-Z_5 + Z_6 - Z_{10} \leq 11.55$	$-Z_7 + Z_8 - Z_{12} \leq 58.80$
$-Z_3 + Z_4 - Z_9 \leq 23.80$	$-Z_5 + Z_6 - Z_{11} \leq 29.05$	
$-Z_3 + Z_4 - Z_{10} \leq 25.55$	$-Z_5 + Z_6 - Z_{12} \leq 39.55$	$Zi \geq 0$

Linear Programming Model for Scenario 2: Locate Transfer Station at T5

The cost function to be minimized is represented by

$$CF = 29.05\,X_{11} + 20.30\,X_{12} + 44.80\,X_{13} + 23.80\,X_{14} + 23.80\,X_{21} + 25.55\,X_{22} + 29.05\,X_{23} + 39.55\,X_{24} + 37.80\,X_{31} + 11.55\,X_{32} + 29.05\,X_{33} + 39.55\,X_{34} + 20.30\,X_{41} + 36.05\,X_{42} + 25.55\,X_{43} + + 50.05\,X_{44}$$

subject to the following constraints:

$X_{11} + X_{12} + X_{13} + X_{14} = 3$	$X_{11} + X_{21} + X_{31} + X_{51} \leq 4$
$X_{21} + X_{22} + X_{23} + X_{24} = 3$	$X_{12} + X_{22} + X_{32} + X_{52} \leq 10$
$X_{31} + X_{32} + X_{33} + X_{34} = 5$	$X_{13} + X_{23} + X_{33} + X_{53} \leq 3$
$X_{51} + X_{52} + X_{53} + X_{54} = 2$	$X_{14} + X_{24} + X_{34} + X_{54} \leq 8$

$$X_{ij} \geq 0$$

Since the cost function needs to be minimized, the constraint equations are first converted to the following Primal equations:

$- X_{11} - X_{12} - X_{13} - X_{14} \geq -3$	$-X_{51} - X_{52} - X_{53} - X_{54} \geq -2$
$X_{11} + X_{12} + X_{13} + X_{14} \geq 3$	$X_{51} + X_{52} + X_{53} + X_{54} = 2$
$-X_{21} - X_{22} - X_{23} - X_{24} \geq -3$	$- X_{11} - X_{21} - X_{31} - X_{41} \geq 4$
$X_{21} + X_{22} + X_{23} + X_{24} \geq 3$	$- X_{12} - X_{22} - X_{32} - X_{42} \geq 10$
$-X_{31} - X_{32} - X_{33} - X_{34} \geq -5$	$- X_{13} - X_{23} - X_{33} - X_{43} \geq 3$
$X_{31} + X_{32} + X_{33} + X_{34} \geq 5$	$- X_{15} - X_{25} - X_{35} - X_{45} \geq 8$

$$X_{ij} \geq 0$$

To convert the problem to a maximization one, the Dual equations become the following:

Maximize: $F = -3Z_1 + 3Z_2 - 3Z_3 + 3Z_4 - 5Z_5 + 5Z_6 - 2Z_7 + 2Z_8 - 4Z_9 - 10Z_{10} - 3Z_{11} - 8Z_{12}$

subject to:

$-Z_1 + Z_2 - Z_9 \leq 29.05$ $\quad -Z_3 + Z_4 - Z_{11} \leq 29.05$ $\quad -Z_7 + Z_8 - Z_9 \leq 20.30$

$-Z_1 + Z_2 - Z_{10} \leq 20.30$ $\quad -Z_3 + Z_4 - Z_{12} \leq 39.55$ $\quad -Z_7 + Z_8 - Z_{10} \leq 36.05$

$-Z_1 + Z_2 - Z_{11} \leq 44.80$ $\quad -Z_5 + Z_6 - Z_9 \leq 37.80$ $\quad -Z_7 + Z_8 - Z_{11} \leq 25.55$

$-Z_1 + Z_2 - Z_{12} \leq 20.30$ $\quad -Z_5 + Z_6 - Z_{10} \leq 11.55$ $\quad -Z_7 + Z_8 - Z_{12} \leq 50.05$

$-Z_3 + Z_4 - Z_9 \leq 23.80$ $\quad -Z_5 + Z_6 - Z_{11} \leq 29.05$

$-Z_3 + Z_4 - Z_{10} \leq 25.55$ $\quad -Z_5 + Z_6 - Z_{12} \leq 39.55$ $\quad Zi \geq 0$

The Dual equations for the sixteen constraint conditions and the objective function F are used in the Simplex method and solution for each Scenario is generated separately.

Computer Application

Since the Simplex method involves matrix iterations to obtain the solution, the fastest way to achieve the solution is to use a computer program. A flow chart representing the computer program structure is given in Figure 2. The program requires the following input data:

- number of transfer stations
- number of disposal sites
- round-trip haul distance between each transfer station and disposal site
- haul speed constants, a and b
- hourly transport cost
- amount of waste delivered to each transport station per day
- daily capacity of each disposal site

For the selected Scenario, the program output provides the amount of waste that can be transported from each transport location to each disposal site in order to minimize the haul costs and the corresponding minimum cost.

Results for the Numerical Example

A program written in Visual Basic is used to solve the numerical example. Table 3 summarizes the results for the two cases of the numerical example: Scenario 1 with transfer station T4 and Scenario 2 with transfer station T5. The minimum operating cost occurs with

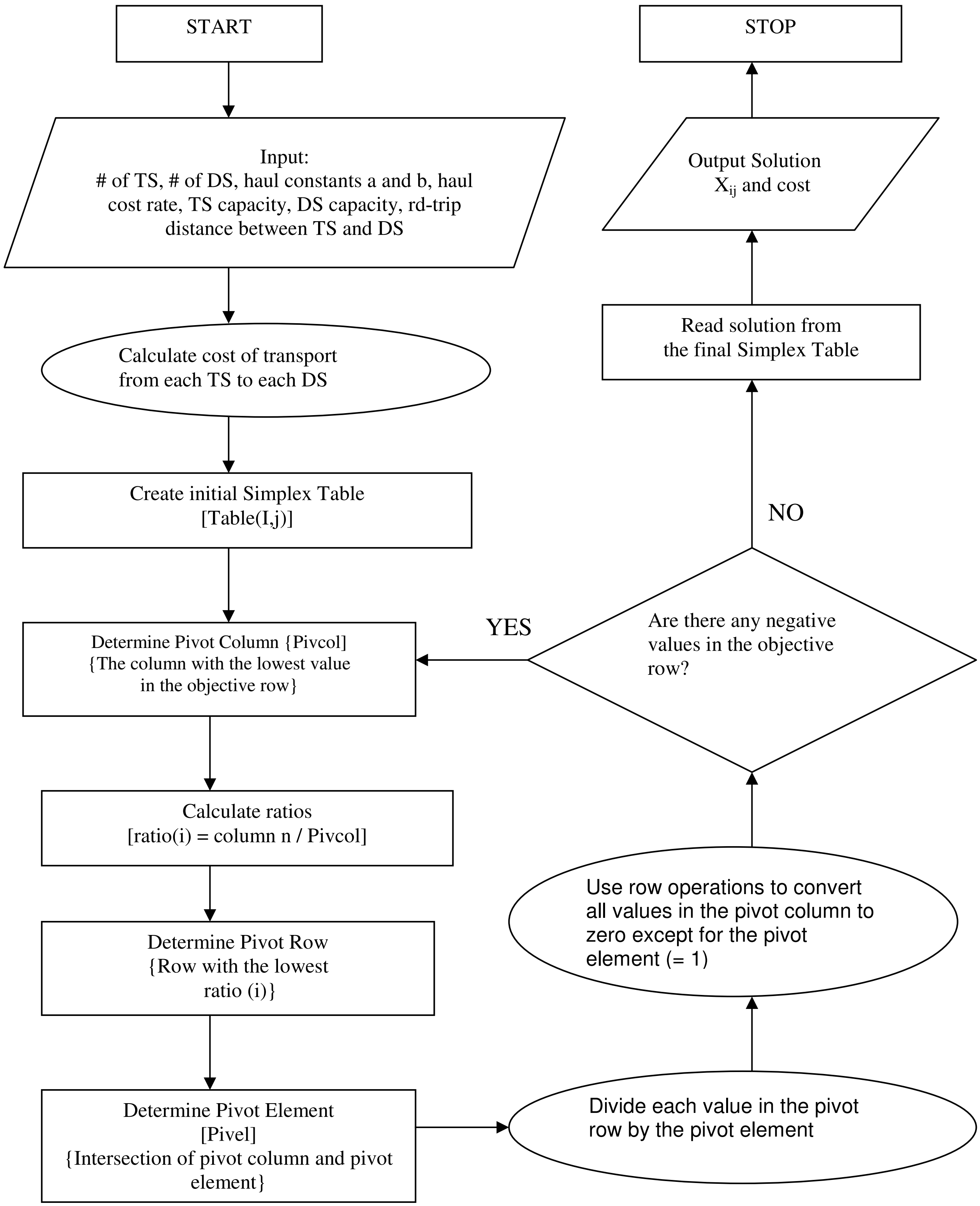

Figure 2. Flow Chart for Computer Code of Simplex Algorithm

Table 3. Computer Program Results for the Numerical Example Comparing Costs for the Two Scenarios.

	Scenario 1				**Scenario 2**			
	Waste Hauled (units/day)				Waste Hauled (units/day)			
	T1	**T2**	**T3**	**T4**	**T1**	**T2**	**T3**	**T5**
D1	0	3	0	0	0	2	0	2
D2	3	0	5	0	3	1	5	0
D3	0	0	0	2	0	0	0	0
D4	0	0	0	0	0	0	0	0
	Total Cost = $209.65 per day				**Total Cost = $232.40 per day**			

transfer station T4 (Scenario 1) compared to transfer station T5 (Scenario 2) and hence the location of T4 will be the site of choice for this example. The total waste amounts of 13 units per day received by the four transfer stations will be distributed to disposal sites D1 (3 units), D2 (8 units) and D3 (2 units) to achieve the minimum cost.

CONCLUSIONS

Linear programming techniques can be effectively used to determine optimum site locations for transfer stations in solid waste disposal operations. With computer application to solve the mathematical model it is possible to obtain fast solutions for large systems with multiple transfer stations and disposal sites.

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