NUMERICAL SIMULATION FOR NONLINEAR STATIC & DYNAMIC STRUCTURAL ANALYSIS

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ABSTRACT

General procedure for numerical simulation is briefly described and applied to realistic structural problems. Analytical structural analysis methods, especially the Finite Element Method (FEM), have become increasingly popular. This paper presents cases of numerical simulation systematically on the three-dimensional nonlinear static and dynamic structural analysis, including reinforced concrete structure and steel frame system with shear wall structure. By comparing with simulation curves, it describes the structural behavior subjected to static loading condition, and evaluates on the serviceability for performance functions with dynamic responses applied in time domain. All analytical data derived from the three-dimensional finite element procedure by ANSYS[®] Software Package.

KEY WORDS

Finite element analysis, nonlinear static & dynamic analysis, reinforced concrete structure, steel frame system with shear wall structure, three-dimensional numerical simulation.

INTRODUCTION

General procedure for numerical simulation is briefly described and applied to realistic structural problems. Analytical structural analysis methods, especially the Finite Element Method (FEM), have become increasingly popular. This is no surprise given the fact that FEM programs are now available on the entire computer hardware platform, ranging from PCs to mainframes, and that FEM has been applied to all engineering disciplines. Modeling, meshing, solving and post-processing have been highly integrated by FEM tools.

There are two proposed structural modeling issues: (I) Reinforced Concrete (RC) beam elastically supported; (II) Steel frame system with shear wall structure. Both of them are implemented by nonlinear static and dynamic structural analysis in this paper. And three-dimensional elements are all defined with ACI & ASTM specifications.

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NONLINEAR ANALYSIS OF FINITE ELEMENT MODEL

Geometric Nonlinearity

Linear structural analysis is based on the assumption of small deformation and linear elastic behavior of material. As applied loads increase, the assumption is no longer accurate, because deformations may cause significant changes in the structural shape. If a structure experiences large deformations, its changing geometric configuration can cause the structure to respond nonlinearly. Newton-Raphson method is used to solve nonlinear problems in the research. Figure 1 illustrates equilibrium iterations of traditional Newton-Raphson method in a single Degree of Freedom (DOF) nonlinear analysis.



Figure 1 Newton-Raphson Method.

Figure 2 Stress-Strain relations of RC materials.

Material Nonlinearity

The reinforced concrete material, initially assumed to be isotropic, is capable of directional integrated points cracking and crushing besides incorporating plastic and creep behaviors. The stress-strain relationships are given in Figure 2 above, with the stress-strain matrix defined as [D] in Equation 1 (Eq.1) below:

$$[D] = (1 - \sum_{i=1}^{N_r} V_i^R) [D^C] + \sum_{i=1}^{N_r} V_i^R [D^r]_i$$
 Eq.1

where: N_r = the number of reinforcement; V_i^R = ratio of the volume of reinforcement.

$$[D^{c}] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix}$$
Eq.2

where: $[D^C]$ = the stress-strain matrix for concrete material, expressed by Equation 2 above, derived from specializing and inverting the orthotropic stress-strain relation to the case of an isotropic material; $[D^r]_i$ = the stress-strain matrix for reinforcement, represented by Equation 3 above, depicted in orientation; E = Young's modulus for concrete material; ν = ratio of the volume of reinforcement; E_i^r = Young's modulus for reinforcement.

As mentioned previously, the reinforced concrete model can predict elastic behavior, cracking or crushing behavior. If the elastic behavior is predicted, it is treated as a linear elastic material discussed above. If the cracking or crushing behavior is predicted, the elastic stress-strain matrix would be adjusted to the failure mode of brittle model, given by Equation 4 below:

$$\frac{F}{f_c} - S \ge 0 \qquad \text{Eq.4}$$

where: F = the function of principle stress state $(\sigma_{xp}, \sigma_{yp}, \sigma_{zp})$; S = the failure surface expressed in terms of principal stress and material parameters; $f_c =$ the uniaxial crushing strength; $\sigma_{xp}, \sigma_{yp}, \sigma_{zp} =$ principal stresses in principal directions.

The stress-strain relation for concrete material while cracking opens in all three directions is expressed by Equation 5 below, furthermore, if cracks in all three directions reclose, the stress-strain relation would be adjusted to Equation 6 below:

$$[D_{C}^{CK}] = E \begin{bmatrix} \frac{R^{t}}{E} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{R^{t}}{E} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_{t}}{2(1+\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta_{t}}{2(1+\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\beta_{t}}{2(1+\nu)} \end{bmatrix}$$
Eq.5

where: R^{t} = the secant modulus in strength of cracked condition.

$$[D_{c}^{\ CK}] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_{c}(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\beta_{c}(1-2\nu)}{2} \end{bmatrix}$$
Eq.6

where: β_{t} , β_{c} = the shear transfer coefficient (0< β_{t} < β_{c} <1).

In addition to cracking and crushing, this model may also undergo plasticity, with the Drucker-Prager failure surface being commonly used. Therefore, the plasticity is done before cracking and crushing are checked out.

NUMERICAL SIMULATION FOR RESEARCH MODELS

Issue I: Reinforced Concrete Beam Models

In this issue, two proposed projects (Project I & II) are created for numerical simulation of three-dimensional nonlinear static and dynamic analysis. The algorithm of Newton-Raphson for an efficient numerical implementation of nonlinear solution strategy, calculates effects of large deformation, plasticity, creep, cracks, internal forces and stresses between different elements in steady loading condition. It is still extended to dynamic analysis in time-varying loading condition.

Models Description

Project I is tested to perform the nonlinear static behavior of reinforced concrete beam elastically supported under steady loads, shown as Figure 3 below. Project II is tested to perform the nonlinear dynamic responses of the same beam model under moving loads, shown as Figure 4 below. Material properties are given in Table 1 below.



Figure 3 Full model in Project I.



Figure 4 Full model in Project II.

No	Poisson'	Young's Mo	dulus (<i>psi</i>)	Material Strength (psi)		Reinforcement
INU	Ratio	Concrete	Reinforcement	Concrete f_c	Reinforcement f_y	Diameter (inch)
Ι	C: 0.2	3.6×10^{6}	2.9×10^{7}	4000	60000 Grade60	#9: 9/8 (29mm)
	R: 0.3	$(2.5 \times 10^4 Mpa)$	$(2 \times 10^5 Mpa)$	(28 <i>Mpa</i>)	(415 <i>Mpa</i>)	#3: 3/8 (9.5mm)
II	C: 0.2	3.6×10^{6}	2.9×10^{7}	4000	60000 Grade60	#9: 9/8 (29mm)
	R: 0.3	$(2.5 \times 10^4 Mpa)$	$(2 \times 10^5 Mpa)$	(28 <i>Mpa</i>)	(415Mpa)	#3: 3/8 (9.5mm)

Table 1: Materials Properties of Reinforced Concrete Beam Models.

With the same elastically supported style in both projects, vertical external loads are acting on the beam model. In Project I, external steady loads are equally divided into two parts. Each part value is defined as F=10.63kip (47.3kN), symmetrically distributed into multipoint nodal loads along the x axis, formed in a line, located with the same distance by 1.5m near the end of beam, detailed in Figure 3 above. In Project II, the constant value of traveling loads is defined as $P=2\times10.63=21.26kip$ (94.6kN), symmetrically distributed into multipoint nodal loads along the x axis, formed in a line during each load step, but moving on the beam with a constant velocity 120 km/h along the z axis, detailed in Figure 4 above. The total value of external loads in each project is definitely the same with the other. In fact, moving loading condition in Project II simulates the realistic complexity of traveling vehicle loads on the reinforced concrete bridge in analysis.

Numerical Simulation for Nonlinear Static & Dynamic Analysis

Both nonlinear static and dynamic responses of structure can be determined through the Newton-Raphson method. The basic simultaneous equation, yielded by the finite element discrete process, is expressed by Equation 7 below:

$$[K] \bullet \{u\} = \{F^a\}$$
 Eq.7

where: [K] = the current stiffness matrix; $\{u\}$ = the vector of unknown DOF values; $\{F^a\}$ = the vector of applied loads. It is an iterative process in solving nonlinear problems. The solution obtained at the end of iterative process would correspond to the load vector $\{F^a\}$. The final converged solution would be in equilibrium. If the analysis included nonlinearities such as plasticity, then the process requires that some intermediate steps reach in equilibrium in order to follow the load path correctly. It is accomplished effectively by specifying a step-by-step incremental analysis, the final load vector $\{F^a\}$ actually is obtained by applying load increments and performing Newton-Raphson iterations in each step.

In Project I, with measured stress-strain relations of the reinforced concrete beam model, geometric and material properties reported above, and principles of FEM procedures in nonlinear static analysis, the simple sketch of calculated deformations by concrete and reinforcing bar are shown as Figure 5 below.

In Project II, the technical transient dynamic analysis is employed to determine dynamic



Figure 5 Displacement translated in Y-axis by RC beam model (Project I).

behavior of the model. The basic equation of transient dynamic solution given by Equation 8:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{P(t)\}$$
 Eq.8

where: [M] = the mass matrix; [C] = the mass damping matrix; [K] = the stiffness matrix;

 $\{P(t)\}$ = the applied load vector; $\{\ddot{u}\}$ = the nodal acceleration vector; $\{\dot{u}\}$ = the nodal velocity

vector; $\{u\}$ = the nodal displacement vector. On the other hand, the full transient method runs with full matrices, furthermore, it allows all types of nonlinearities to be included, such as large deformations, etc. Newmark assumptions are considered with Newton-Raphson method. Since the traveling loads of *P* (with the constant velocity 120 km/h along the *z* axis.) excite all modes of the model, dynamic responses have been tracked. This project employed 15 load steps with 5 sub-steps in each one during computational simulation processing. Deformations in the 2nd and 8th load step are more characteristic, shown as Figure 6 below.





Distributions on peak values of nodal displacement responses are not symmetrical while traveling loads are moving on the beam with a constant velocity in Project II. Figure 7 shows two groups of nodal displacement responses in time domain. Each group is made up of two characteristic nodes located symmetrically along the length of beam model.





The largest nodal displacement and velocity responses occurred in the 8th load step, shown as Figure 8. Simulation of internal stresses and cracks are implemented as Figure 9.



Figure 8 Displacement (L) & velocity (R) responses in the 8th load step by RC beam model.



Figure 9 Simulation of reinforcing bar element stress & RC beam concrete cracks.

Issue II: Steel Frame System with Shear Wall Structure Models

Many government agencies and some private building owners today require that new buildings be designed and existing buildings upgraded to resist the effects of potential blasts. Although the probability that any building actually would be subject to such hazard is low, a performance-based analysis with a visible comparison between the nonlinear static and

dynamic behaviors of building structures has made a contribution to optimize future design.

Similarly, this is another issue of two proposed projects (Project III & IIII) created for numerical simulation of three-dimensional nonlinear static and dynamic analysis. Testing determined on the steel frame system with shear wall structure models. Some fundamental algorithms of nonlinear solution strategy are still the same with issue I. However, with the identification by different material properties and different element types, the finite element model procedures must be changed.

Models Description

As the original frame system with shear wall structure had been designed for gravity load, it proved to attempt to resist in current dynamic blast-impulsive loading condition. Project III is tested to perform the nonlinear static behavior of steel frame system with shear wall structure under steady loads. Project IIII is tested to perform the nonlinear dynamic responses of the same structural model under dynamic blast-impulsive loads with transient dynamic analysis. This frame system is modeled by two-storey two-bay in each direction. Rigid multipoint constraints are used to enforce the rigid stiffness at bottom. Besides gravity load is applied, horizontal and vertical external nodal loads are defined as H=50kip (222kN) and F=150kip(667kN). Moreover, each value of nodal loads in corresponding node in each project is definitely equal, but acting with absolutely different modes, shown as Figure 10. Material properties are given in Table 2 below. Steel girders and columns are defined as A992, reinforced concrete slabs and shear wall are defined by a compressive strength of 6000*psi*.



Figure 10 Full model of Issue II (Project III & IIII).

Table 2: Properties of	f Steel Frame System	with Shear Wall	Structure Models.
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No	Poisson' s	Size o	f Steel Steel Young's		s Modulus (<i>psi</i>)	Thickness of RC (inch)	
	Ratio	Beam	Column	Steel Frame	RC system	Floor	Wall
III	S: 0.3	$W_{10} \vee c_0$	W12×58	2.9×10^{7}	4.4×10^{6}	5.5	5
	C: 0.2	W18 ^ 00		$(2 \times 10^5 Mpa)$	$(3.0 \times 10^4 Mpa)$	(140 mm)	(127 mm)
IIII	S: 0.3	$W_{10} \times c_0$	W12×58	2.9×10^{7}	4.4×10^{6}	5.5	5
	C: 0.2	W18×60		$(2 \times 10^5 Mpa)$	$(3.0 \times 10^4 Mpa)$	(140 mm)	(127 mm)

Numerical Simulation for Nonlinear Static & Dynamic Analysis

The solution algorithm of Newton-Raphson method has been used in the simulation process. All loads in Project III are acting on the model within 1 load step for nonlinear static analysis. To simulate the dynamic blast-impulsive nodal loads in Project IIII, totally 7 load steps adopted during the full transient dynamic analysis with time integration scheme. The basic equation of Eq.8 can be thought of as a set of "static" equilibrium equations that also take into account inertia forces and damping forces. Newmark time integration scheme is used to solve these equations at discrete time points.

In the post processing, Figure 11 put together with the calculated deformations in Project III & IIII, and both main nodal displacement responses of the central column in time domain are shown as Figure 12. Figure 13 put together with the nodal stress distributions in Project III & IIII, shown as below. It makes a visible comparison with nonlinear static and dynamic behaviors of the same structural model.

Actually, when resisting direct air-blast loading can produce high strain rates, perhaps of larger magnitude, and will occur simultaneously with large axial tension demands. In this condition, steel frame system becomes stronger but more brittle. There is evidence that indicated standard beam-column connection types used in steel frame system might not be



Figure 11 Nodal Displacement Responses by Project III & IIII.



Figure 12 Nodal Displacement Responses in Time Domain by Project III & IIII.



Figure 13 Nodal Stress Responses by Project III & IIII.

capable of allowing structure to develop the large inelastic rotation and tensile strains. Whereas, when properly configured and constructed using materials with appropriate toughness, steel connections can provide outstanding ductility and toughness.

CONCLUSION

A general goal of this paper on nonlinear solution is to avoid severe damage in various complex loading conditions. The applications of two proposed modeling issues including 4 projects on cases of reinforced concrete structure and steel frame system with shear wall structure subjected to static and dynamic loads are implemented by ANSYS[®] Software Package using finite element method in this paper. The comparison with simulation curves are represented to optimize structural design.

Computational simulation of nonlinear static and dynamic structural analysis based on advanced Three-Dimensional Finite Element (3DFE) models can be effectively used for an efficient investigation of realistic structural behaviors in different loading conditions, for a good solution strategy to solve technical problems. In fact, the powerful FEM programs have established a set of rules for structural behaviors in numerical simulation.

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