# POSITIONING FEM IN THE TRANSFORMATION OF SPATIAL DESIGN TO STRUCTURAL DESIGN

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*ABSTRACT: In the field of architecture, structural topology is the set of locations, types (i.e. beams, columns), and arrangements of structural elements. To help the architect in understanding the structural topology for his spatial design, research exists that studies the process of the transformation of a spatial design into a structural design. If the Finite Element Method (FEM) is applied in this research, two problems can occur: (1) how to transform a topology in a mechanical system and FEM-input and (2) how can FEM support qualitative design decisions. In this paper, it is tried to define these two problems more clearly by developing data(EXPRESS)- and process(IDEF0)-models for three transformations: From structural topology to mechanical system, from mechanical system to finite element model, and from finite element model to design recommendations. A six-level apartment building is used as a case study to test and supplement the data- and process-models for all three transformations. It can be concluded that the data- and process models are useful at their abstract level, but that many problems at lower abstraction levels remain to be solved.*

*KEYWORDS: FEM, structural design, spatial design, data model, process model.*

## 1 INTRODUCTION

In the field of Architecture, Engineering, en Construction (AEC), most design processes are multi-disciplinary and many research projects are carried out to investigate these design processes and to develop (computer aided) tools to support the processes in order to make them more efficient (Haymaker *et al* 2004, Fenves *et al* 1994, Fenves *et al* 2002, Mora *et al* 2006, Khemlani *et al* 1998, Matthews *et al* 1998, Rosenman *et al* 2005, Hofmeyer and Kerstens 2006, Hofmeyer 2007). Furthermore, researchers assume that by improving the design processes, the quality of the designed product (the building) improves as well. In this paper, within the multi-disciplinary design process, only the disciplines of spatial design and structural design will be considered.

Research on the computational aspects of spatial design and structural design can be divided in two groups: space-allocation (i.e. Kotsopoulos 2005, Reffat 2006, Keatruangkamala and Nilkaew 2006, Oxman 1997) and structural optimization & grammars (Maher 1985, Mullins *et al* 2005, Bletzinger and Ramm 2001). Within these groups, often the basic underlying idea on the design process is that a more or less one-way path runs from spatial to structural design (figure 1a).

However, the building design process can also be modelled with a more cyclic approach. A start is made by the transformation of a spatial design into a structural design, which is carried out often by a structural designer. The resulting structural design will be subject for improvement, for example by expert views of other structural designers or by optimization techniques. This optimised structural design will be given to the architect and he will then adjust the spatial design to fit the structural design (step 3, from structural design 2 to spatial design 2) or to fulfil other requirements from the building plan (step 4, from spatial design 2 to spatial design 3).

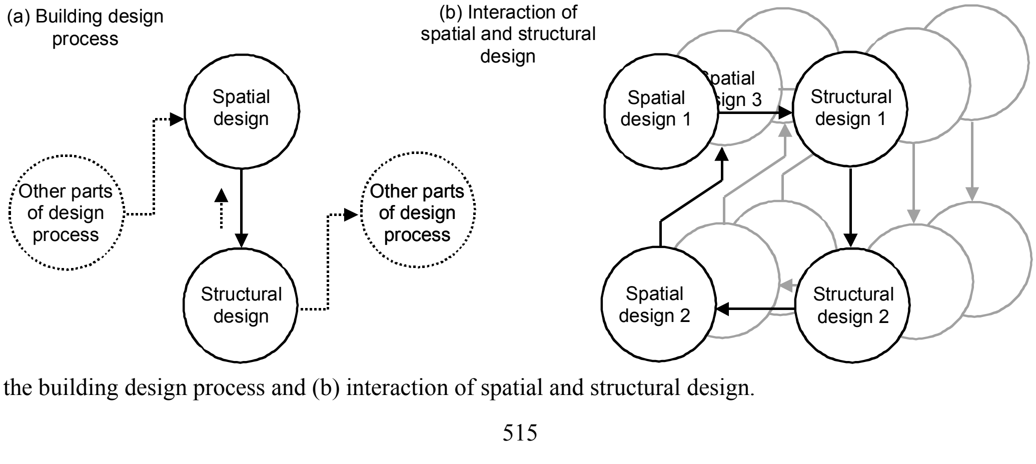



Figure 1. (a) the building design process and (b) interaction of spatial and structural design.

The resulting design spiral -as shown in figure 1b- is defined as "interaction between spatial and structural design" and the use of this model of the design process is justified by many research projects on the support of multi-disciplinary design processes, for example (Haymaker *et al* 2004) who show that a building design project can be seen as a sequence of views and dependencies from several disciplines.

Recently, it was shown that a procedure for cyclic transformations between spatial and structural designs with the use of a scale to evaluate design characteristics yields fundamental knowledge on both interaction between spatial and structural design, and the underlying design process (Hofmeyer 2007). The research was driven by the idea of a research engine as shown in figure 2. By applying selected transformations and evaluating the degree of inter-disciplinary design by using a scale, the fundamental relationship between spatial and structural design can be investigated.

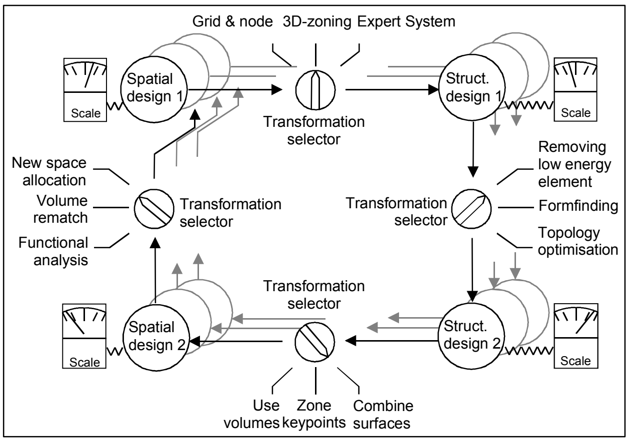



Figure 2. Research engine.

In both research groups previously mentioned (space-allocation and structural optimization & grammars) and in the last mentioned research on cyclic transformations, the transformation of a spatial design into a structural design is an important aspect. Often, the finite element method is applied for parts of this transformation and then the following two problems can occur (1) how to transform a topology (the set of locations, types (i.e. beams, columns), and arrangements of structural elements) into a mechanical system and FEM input and (2) how can FEM support qualitative design decisions. In this paper, the two problems mentioned are defined in a more clear way by defining specific data- and process-models, based partly on general data-models for spatial and structural design (Mora *et al* 2006) and general process-models for spatial and structural design (Sause *et al* 1992, Sacks and Warszawski 1997). Three stages are considered. In section 2 the process from structural topology to mechanical system, in section 3 from mechanical system to finite element model, and in section 4 from finite element results to design recommendations. Then in section 5, these data- and process-models are used for the design process of a six-level apartment building and problems found here will be reported in terms of the data- and process-models. Throughout the paper, only linear elastic behaviour is considered, thus buckling and other stability problems are not taken into account.

## 2 FROM STRUCTURAL TOPOLOGY TO MECHANICAL SYSTEM

To define the structural topology of a design, several models exist. In this paper, the StAr data-model of Mora *et al* 2006 will be used, because this is the most extended and useful model at the moment. In this paper, the StAr's architectural model serves as structural topology. This topology, a number of spatial elements Abeams, Acolumns, Aslabs, Awalls, and their position and properties, is not suitable for structural calculations. This because the architectural elements often will not form a kinematically determined and fixed "mechanical system". A mechanical system consists of mechanical elements, their properties, dynamic boundary conditions, and kinematic boundary conditions, figure 3.

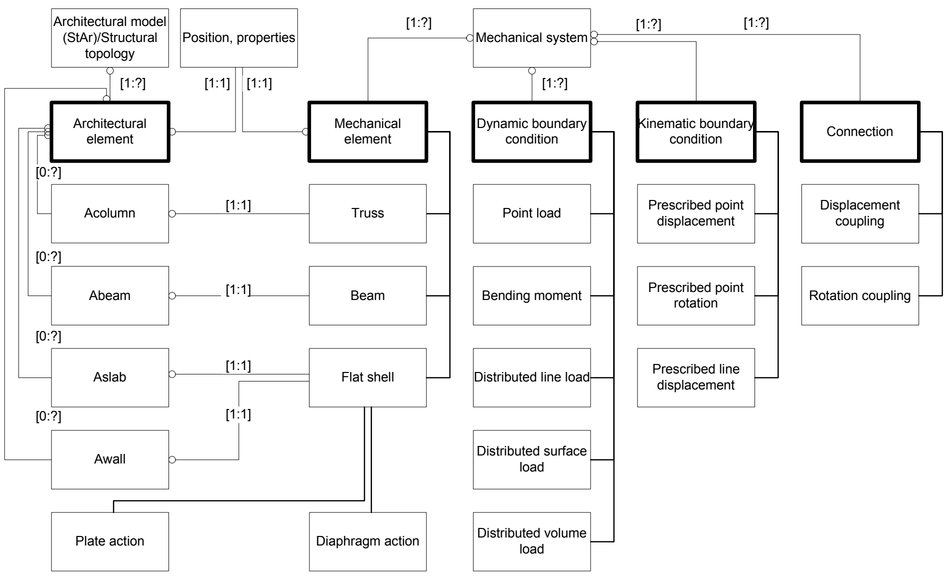



Figure 3. Data-model for the transformation of a structural topology into a mechanical system, using the EXPRESS (Schenk and Wilson 1994) representation.

The mechanical elements are of type truss (a line element only able to resist normal forces), beam (a truss that also resists bending and torsional moments), and flat shell. A flat shell has both diaphragm action (resisting in-plane loading by membrane forces) and plate action (resisting out-of-plane loading by bending and torsional moments). Note that normally in a mechanical system, depending on the load type and orientation, diaphragms and plates exist as separate elements, but to reduce complexity in this paper they are combined. The same is valid for volume elements, these are omitted here. Every mechanical element should be connected to at least one other element such that it cannot displace or rotate freely relative to this element. For this, two types of connections can be used, displacement and rotation couplings. Furthermore, the mechanical system itself, containing all connected mechanical elements, should not displace or rotate relative to the earth. Therefore kinematic boundary conditions are needed, the most frequently used types are a (zero) prescribed displacement or rotation for a point and a (zero) prescribed line displacement. Loading the mechanical system will be carried out by dynamic boundary conditions, such as a point load, bending moment, and distributed line or surface load.

Given the data of structural topology and mechanical system in figure 3, the process to transform a topology into a mechanical system should be developed. To start with, it seems logical to generate a corresponding mechanical element for every architectural element, corresponding sets can be found in figure 3. The most difficult task is then to generate additional mechanical elements or connections, such that the set of mechanical elements becomes a mechanical system. For this, first it will be defined when a mechanical element is fixed in space, given the following assumptions:

- A mechanical element can only have kinematic boundary conditions at its element points. Element points are defined as the ends (truss or beam) or the corners (flat shell).
- Only translational kinematic boundary conditions will be taken into account. This means that the system can only become kinematically determined by shear walls (flat shells) or bracings.
- Only designs are regarded that have a more or less rectangular setting. This does not mean that only box-like buildings can be used, but for instance so-called blobs and tensegrity-structures are out of scope here.

Given these assumptions, figures 4 (a) and (b) show the minimal conditions for mechanical elements to be fixed in space, using a local coordinate system for the element. The procedure to develop a mechanical system could now be as follows. Given a global coordinate system, the set of element points having the same and lowest *y*-value (in other words are at ground/foundation level) are constrained for displacement along the *x*-, *y*-, and *z*-axis, step 1 in figure 4 (c). If this set consists of only points along a single line, than additional elements should be generated such that the points do not lie along a single line. Then one of the involved elements is investigated for the minimal conditions. If not all conditions are met, additional conditions are found by connecting the element points to other element points of elements already fixed or to truss or beam elements fixed at least in their normal direction. In this case of the example in figure 4(c), flat shell 1 needs an additional constraint for the *z*-displacement, and this can be found by adding a truss, step 2 in figure 4(c). This procedure is then followed for all mechanical elements. For the example, flat shell 1 (diaphragm action) can now be regarded as fixed in space and the adjacent part flat shell 2 is checked for the minimal conditions. For flat shell 2 an additional *y*-constraint is necessary. Because it is connected to a truss in *y*-direction and this truss is already fixed in *y*-direction, the flat shell (plate action) is also fixed. Then the already placed truss is fixed too, step 3. In the last part of the example, the truss on top is fixed by two additional truss elements, step 4. These processes are modelled using IDEF0 as shown in figure 5 and 6. Hereafter, the properties of the mechanical elements have to be defined, i.e. material properties and cross-sectional parameters. For the material properties, it is assumed that the architectural model already defines the material and that these properties are inherited by the mechanical element. Assuming linear elastic behaviour, only Young's modulus $E$, Poison's constant $\nu$, and the density $\rho$ are used. For cross-sectional parameters, conceptual structural design rules are used (e.g. Lin and Stotesbury 1981). This is acceptable, as (cross-sectional) properties are only used for a rough estimation of the strains and stresses in order to move, remove, or add elements (section 4) and not specifically to optimize the elements themselves. The process of loading the mechanical system consists of three parts, namely applying the gravitational load, lateral load, and live load, as also follows indirectly from the StAr-representation (Mora *et al* 2006).

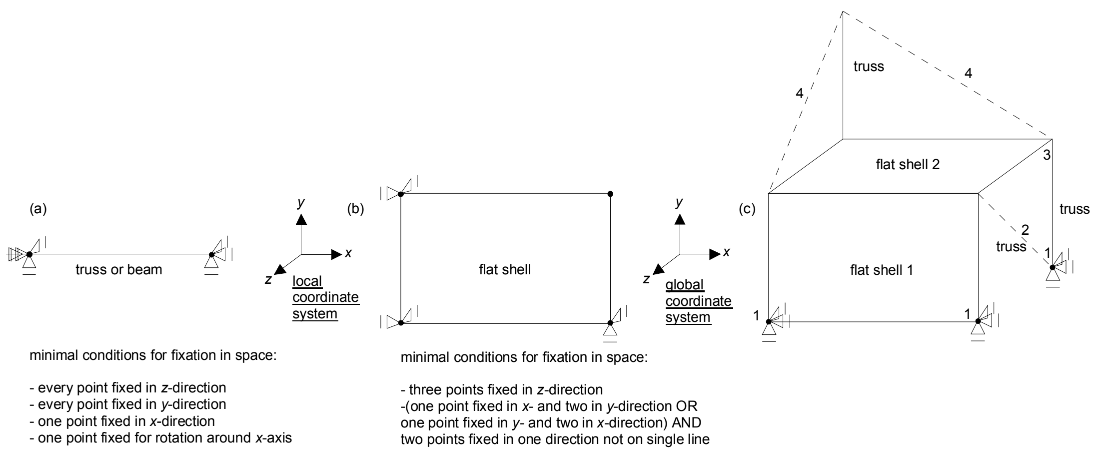



Figure 4. (a) and (b) conditions for elements to be fixed, (c) example.

Given the fact that the mechanical model is to be transformed into a finite element model, the gravitational load is the most easy to apply. This because in the finite element model, if the material and thus density of a mechanical element are known, gravitational loading can simply be switched on. Live load can be applied in the mechanical model by finding all horizontal flat shells and applying a distributed load of 2.0 kN/m$^2$. Lateral loading is applied by first generating side views of the building, and then applying vertical distributed loads of 1.0 kN/m$^2$ on all flat shells in line of sight.

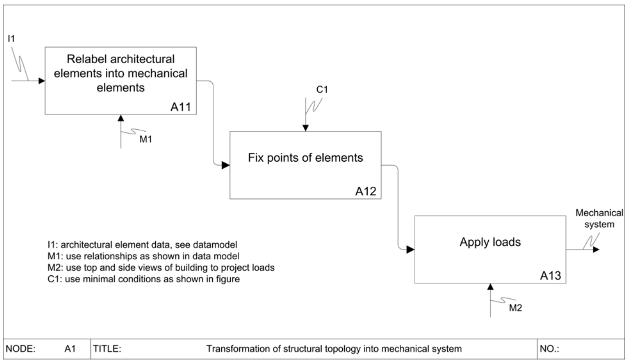



Figure 5. Process-model for topology into mechanical system using IDEF0 (Knowledge Based Systems 2007).

Although these data and process models make sense at their level of abstraction, it should be noted that for many processes on a lower abstraction level no solutions can be found in literature and solutions may be very hard to find. For instance, if process A12 (Fix points of elements, figure 6) is carried out, at least two problems occur: A mechanical system is developed that may have unpractical connections between mechanical elements, for instance a beam may cross a corridor. Or, the connections that are added to the system to make it kinematically determined are not logically from a structural point of view regarding stiffness or strength. It should be investigated whether a cyclic application of transformations, having spatial considerations, and presented in the introduction and figure 2, could filter out this sort of problems.

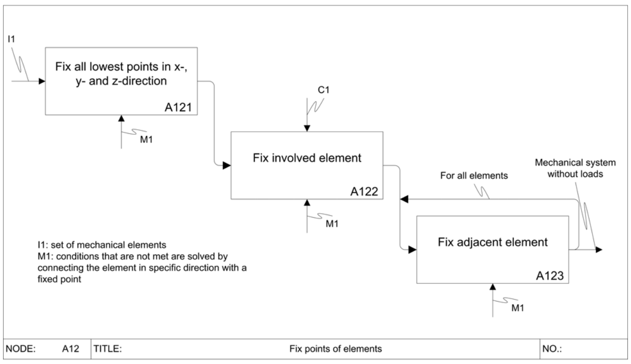



Figure 6. Process-model for fixing of element points using IDEF0 (Knowledge Based Systems 2007).

# 3 FROM MECHANICAL SYSTEM INTO FINITE ELEMENT MODEL

The developed mechanical system still is not a finite element model and in general many problems may occur if a finite element model has to be developed (Bakker and Pekoz, 2003). Figure 7 shows a proposed data model of the mechanical system and a finite element model. In the transformation of a mechanical system into a finite element model, the most difficult part is that a mechanical element is normally not directly modelled by a finite element, but is seen as a geometrical entity (keypoint, line, or surface) and that a geometrical entity is modelled by finite elements. "Line element division" is used to determine how many finite elements are used for one geometrical entity. Depending on the stress gradient in a specific part of the mechanical system, a minimal number of elements is needed for good results, the use of too many elements is regarded as inefficient. Another aspect that complicates the choice for the number of elements is that this number influences a structural optimisation process in which low-stress elements are removed from the finite element model (Hofmeyer, 2007). For the process from mechanical system into finite element model, it is proposed to start with generating one geometrical entity for each mechanical element. Then each line of the geometrical entity is divided by "Line element division" in a number of elements that corresponds with the expected stress gradients. For FE Truss and Beam elements, one element is used for each line. For flat shells, 3 * 3 4-node elements will be used. Dynamic and kinematic boundary conditions are modelled by loads like displacements, force or moments, and pressures. In general, connections need not to be modelled as finite elements couple their degrees of freedom (DOF) at coincident nodes. However, also rotational DOF's will be coupled by the Finite Element Method (FEM) which is not congruent with the approach in section 2 where only translational DOF's were considered. Because it is quite difficult to set up the Finite Element model such that only translational DOF's are applied, for now the method in section 2 is used as a minimal condition for a kinematically determined design, whereas the additional DOF's applied by FEM are regarded as providing additional stiffness. It should also be noted here that the finite element method may provide an alternative for the procedures in section 2 to make the mechanical system structurally kinematically determined. As long as the system is not kinematically determined, a finite element model will not work (regrettably some finite elements programs use a work-around that prohibits the user to see this problem) and thus it may be possible to add mechanical elements in an iterative method until the program does work (and the system is kinematically determined).

# 4 FROM FINITE ELEMENT MODEL TO DESIGN RECOMMENDATIONS

Once a finite element model is developed, it can be used for, in this paper, a linear elastic calculation. In fact, the only direct output of the finite element model is a set of reaction forces and displacements of the element nodes, figure 8. Using the nodal displacements then, strains and stresses in the finite elements can be predicted. In this paper, the finite element results are not used for precise design of section properties or exact prediction of the strength of elements or systems, but are merely meant for design recommendations in the conceptual design phase.

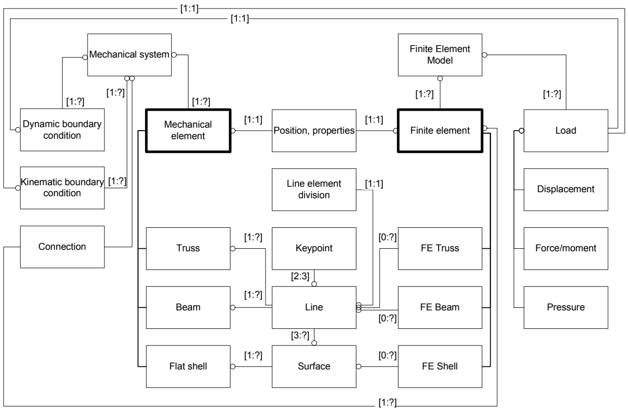



Figure 7. Data-model for the transformation of a mechanical structure into finite element model using the EXPRESS (Schenk and Wilson 1994) representation.

These design recommendations could be:

- A mechanical element should be supported by an additional element to increase the strength and/or stiffness of the mechanical system.
- A mechanical element can be removed without affecting the strength and/or stability of the mechanical system.
- A mechanical element should be moved (position) or added to optimise the strength and/or stiffness of the mechanical system.
- The mechanical element's properties (material properties or cross-section parameters) should be changed to optimise the strength and/or stiffness of the mechanical system. Although the element's properties are needed for carrying out a useful finite element calculation, this fourth design recommendation is not used in the paper, because as discussed the recommendations are merely meant for the conceptual design phase.

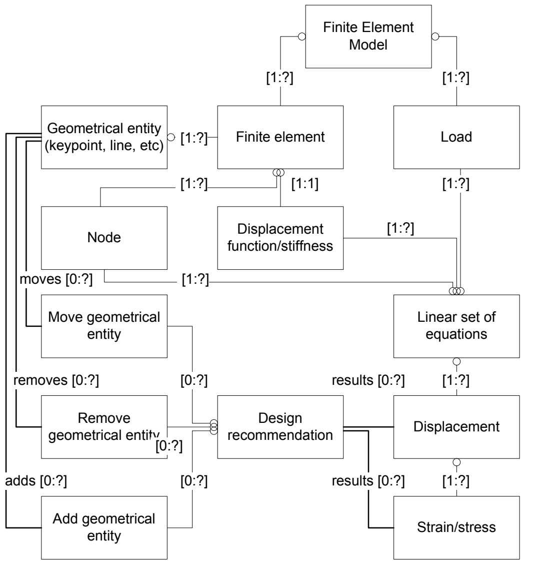


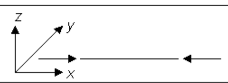
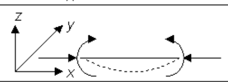
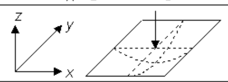
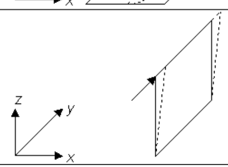

Figure 8. Data-model for the transformation of finite element results into design recommendations, using the EXPRESS (Schenk and Wilson 1994) representation.

For stiffness considerations, table 1 shows for each element for which finite element output ("displacement" and/or "strain/stress" in figure 8) a design recommendation would be appropriate. Note that a similar table could be made for strength considerations, but in this paper, only stiffness will be considered. It is assumed that every element is fixed in space (by the procedures as presented in figure 5), thus large (rigid body) displacements are no valid argument to give a design recommendation.

## 5 CASE STUDY

The developed strategies to transform a structural topology into a finite element model (with a mechanical system) and to interpret the finite element results to optimize the structural design have been tested for the spatial design of a six-level apartment building as shown in figure 9. All three processes (in section 2, 3 and 4) were carried out manually, whereas the FE simulations were, of course, carried out automated. Conclusions of this case study can be used to refine the data- and process-models and these can then be automated, including the design recommendations (section 4).

Table 1. Conditions distilled from finite element output for design recommendations.

| Mechanical element | | Element should be supported | Remove element (from structural point of view) | Element should be moved or added |
|---|---|---|---|---|
| Truss |  | Large normal deflections | Very low strains in normal direction | Far adjacent elements have larger displacements in truss normal direction than elements directly adjacent |
| Beam |  | Large bending deflections | Very low bending and normal strains | Far adjacent elements have larger displacements in direction perpendicular to beam than elements directly adjacent |
| Flat shell (plate action part) |  | Large out-of-lane deflections | Very low bending strains | |
| Flat shell (diaphragm action part) |  | Large in-plane deflections | Very low normal strains | Far adjacent elements have larger displacements in flat shell direction than elements directly adjacent |

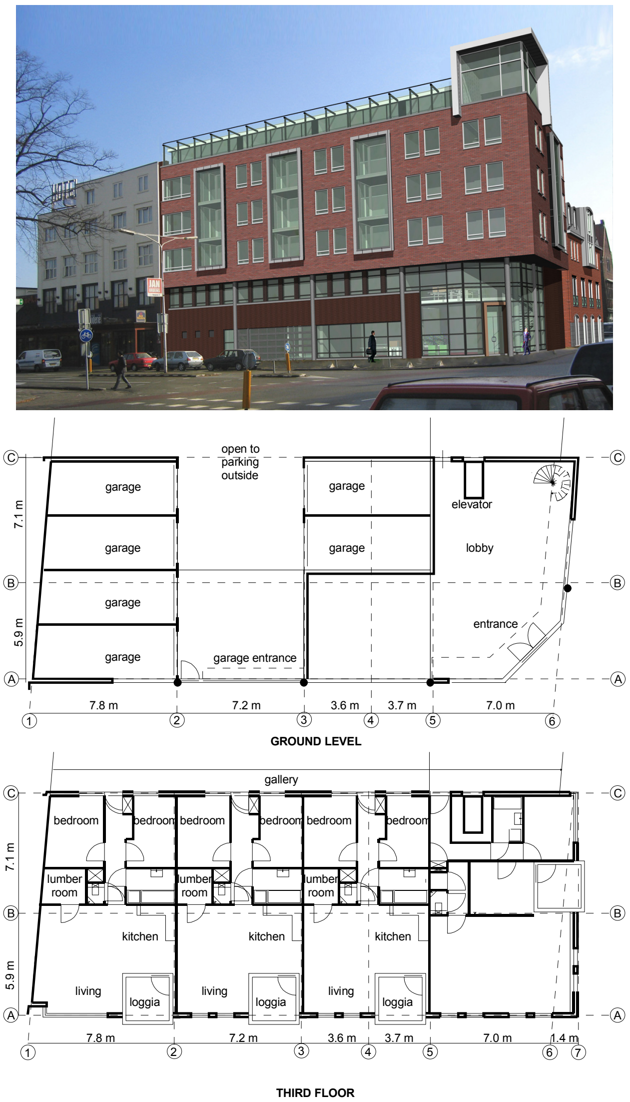



Figure 9. Spatial design (Sturm Architects, Roosendaal, The Netherlands) of a six-level apartment building.

Regarding the design, the ground floor is mainly used for parking and provides the entrance and lobby. From the second floor on apartments are planned. The third to fifth floor form a single architectural block as seen from the outside (see the artist impression in figure 9 at the top). This block seems -in the architects opinion- to be supported by six columns visible at the outside over the first two levels. The deviant structure of the upper deck (the sixth floor) is out of the scope of this paper. As a first step, the structural designer involved in this project was interviewed. This interview can be used to verify the proposed strategy to transform a structural topology into a mechanical system: After the structural designer received the architectural drawings, he first selected vertical positioned elements (such as walls and columns) that could be used as elements in a mechanical system. He carried out this selection by choosing all columns shown in the drawing (this because "a column is always a structural element") and selecting walls that were cavity walls. Following this selection process, he investigated whether the floors were sufficiently supported and whether forces could flow from the top of the building to the basement following a more or less straight vertical line. Table 2 shows the differences between the proposed method and the findings in the interview. In general, it can be stated that the (automated) proposed method is more error-prove, but possibly does not recognise optimal or intuitive acceptable solutions.

Table 2. From topology into mechanical system, proposed method vs. case study.

| Issue | Proposed method (section 2) | Structural designer (case study) |
|---|---|---|
| Transformation | Generate a mechanical element for each architectural element | Structural designer uses all Acolumn's, selects Awall's that are cavity walls, uses all Aslab's. |
| Fixation/stability | Using minimal conditions (fig. 4), fixing element points in a systematic way. | Investigation whether floors are supported sufficiently (for stiffness, not for stability). Availability of diaphragms in two perpendicular walls. |
| Optimization | Is thought to take place in the research engine (fig. 2), not at this step | Studies whether forces flow in a more or less straight vertical line |
| +/- | (+) A kinematically determined structure is always found<br>(-) All architectural elements are selected, not useful for transparent parts<br>(-) No optimization | (-) Possibly no kinematically determined structure<br>(-) Intuitive selection of elements not easy to program<br>(+) Optimization |

The structural designer selects architectural elements whereas the proposed method uses all architectural elements. Thus the mechanical system by the structural designer is a subset of the system by the proposed method. Because a subset may be more demanding in finding a structural system, this subset is chosen for the following step "from mechanical system into a finite element model". Figure 10 shows the finite element model with the stress-distribution due to lateral and live load. The following items are subject to improvement:

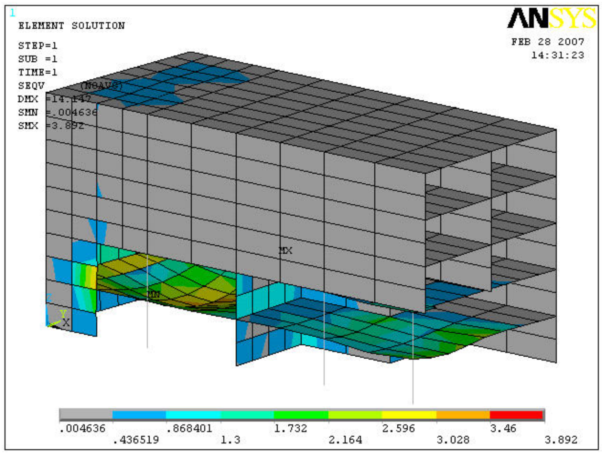
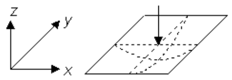
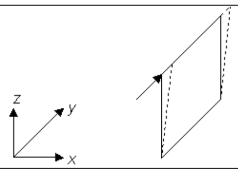



*Figure 10. Finite simulation of six level apartment building. At the right a cross-section has been made by removing elements.*

If mechanical elements do not end at but do join other mechanical elements, node connectivity is not guaranteed. This problem occurs for instance if the Awall at gridline 1 (figure 9) is defined as positioned from gridline A to C. If three finite elements are used along the length (as suggested in this paper) the element nodes are not coincident with the element nodes of the perpendicular walls. Thus the mechanical elements have to be divided into smaller elements that make node connectivity possible. For floors, it became clear

Table 3. Design recommendations case study.

| Mechanical element | | Element should be supported | Element can be removed | Element should be moved or added |
|---|---|---|---|---|
| Truss |  | No, normal deflections are ok | No, all trusses are needed | Two columns (trusses) could be moved a little to support the floor better |
| Flat shell (plate action part) |  | Yes, figure 10 shows some severely deflected areas | No, all shells are needed | |
| Flat shell (diaphragm action part) |  | No, normal deflections are ok | Yes, within the building (see cross-section figure 10) some elements can be removed | Yes, some of these elements could move to ground level to support the floor. |

that mechanical elements both beneath and above the floor have to be considered for divisions of Afloor.

1. If side views of the building are used to apply lateral loads, Awalls that were not taken into account as mechanical elements, such as a glass facade or a frame with cladding, do not show up. This means they are not loaded, and thus it may be a significant amount of load that is not applied. The same is valid for dead load of architectural elements not taken into account within the mechanical system.
2. The amount and location of architectural elements was such, that the derived mechanical system was kinematically determined without further addition of elements. This suggests again that architects unconsciously take structural design aspects into account (Hofmeyer 2007).
3. For floors it is needed to know their span direction. Now in the finite element simulation, a floor was used that spans in both directions, but this is often far from realistic.

For the last step table 1 was used to provide recommendation on an optimised structural design, table 3. This would lead to a new structural design as is shown in figure 2 (structural design 2).

## 6 CONCLUSIONS

The transformation from spatial design into structural design has been split up into three parts: (1) from spatial topology into mechanical system, (2) from system into finite element model, and (3) from finite element model to design recommendations.

For all three parts, data-models have been developed using EXPRESS. For the first part, also a process-model was made using IDEF0 (for the other two parts, processes were only described in text). This modelling was not possible without limiting the types and number of boundary conditions, and the exclusion of freeform structures like blobs and tensegrity structures.

The data- and process-models have been verified with the design of a six-level apartment building as case study. This verification shows that mechanical elements have to be split to guarantee node compatibility, that architectural elements not included into the mechanical model cause lateral- and live-load errors, and that span directions of floors should be investigated. Furthermore, also rotational DOF's are coupled by the Finite Element Method (FEM), which is not congruent with the approach in section 2 where only translational DOF's were considered. The assumption has been made that (cross-sectional) properties are only used for a rough estimation of the strains and stresses in order to move, remove, or add elements, not specifically to optimize the elements themselves. And finally, only stiffness, not strength, was considered in the design recommendations.

Although many assumptions and limitations were taken, the application of the data- and process models showed that they are useful at their abstract level. However, many problems at the lower abstraction levels remain to be solved. For instance, in section 2 it was shown that unpractical new mechanical elements between existing ones can be generated or connections cannot positioned logically from a structural point of view. It should be investigated whether a cyclic application of transformations, as presented in the introduction and figure 2, could filter out this sort of problems.

## REFERENCES


Bakker, M.C.M., and Pekoz, T. (2003). "The finite element method for thin-walled members--basic principles" Thin-Walled Structures 41(2-3) 179-189.

Bletzinger, K.U. and Ramm, E. (2001): "Structural optimization and form finding of light weight structures" Computers & Structures 79 2053-2062.

Fenves, S., Flemming, U., Hendrickson, C., Maher, M.L., Quadrel, R., Terk, M. and Woodbury, R. (1994) Concurrent Computer-Integrated Building Design, PTR Prentice Hall, Englewood Cliffs, New Jersey.

Fenves, Stephen J., Rivard, H., and Gomez, N. (2002) "SEED-Config: a tool for conceptual structural design in a collaborative building design environment" Artificial Intelligence in Engineering 14 233-247.

Haymaker, J., Fischer, M., Kunz, J. and Suter, B (2004). "Engineering test cases to motivate the formalization of an aec project model as a directed acyclic graph of views and dependencies" ITcon 9 419-441.

Hofmeyer, H., Kerstens, J.G.M. (2006) "Full 3D Structural Zoning of Space using a Geometrically Related Reducer and Matrix Coupling" Atsuko Kaga; Ryusuke Naka: CAADRIA '06, Proceedings of the 11th Conference on Computer Aided Architectural Design Research in Asia, March 31-April 2, Kumamoto, Japan, ISBN 4-907662-01-7, 161-168.

Hofmeyer, H. (2007). "Cyclic application of transformations using scales for spatially or structurally determined design"

Automation in Construction, In Press, Corrected Proof, Available online 2 January 2007 (http://www.sciencedirect.com).

Knowledge Based Systems (2007). "IDEF0, Integrated DEFinition Models" Knowledge Based Systems, Inc., College Station, Texas Corporate Headquarters, One KBSI Place, 1408 University Drive, East, College Station, TX 77840, USA, http://www.idef.com/pdf/idef0.pdf

Keatruangkamala, K., and Nilkaew, P. (2006). "Strong Valid Inequality Constraints for Architectual Layout Design Optimization", in: Kaga, A and Naka, Ryusuke, eds., CAADRIA '06 Proceedings of the 11th Conference on Computer Aided Architectural Design Research in Asia, CAADRIA Morozumi Laboratory, Kumamoto University, 179-185.

Khemlani, L., Timerman, A., Benne, B., and Kalay, Y.E. (1998). "Intelligent representation for computer-aided building design" Automation in Construction 8 49-71.

Kotsopoulos, S.D. (2005) "Constructing Design Concepts, A Computational Approach to the Synthesis of Architectural Form", Ph.D.-thesis Massachusetts Institute Of Technology (MIT), USA

Lin, T.Y., Stotesbury, S.D. (1981) Structural Concepts and Systems for Architects and Engineers, John Wiley & Sons.

Maher, M.L. (1985). "HI-RISE and beyond: directions for expert systems in design" Computer-Aided Design 17(9) 420-427.

Matthews, K., Duff, S. and Corner, D. (1998) "Model for Integrated Spatial and Structural Design of Buildings" CAADRIA '98 [Proceedings of The Third Conference on Computer Aided Architectural Design Research in Asia / ISBN 4-907662-009] Osaka (Japan) 22-24 April 1998, 123-132.

Mora, R., Rivard, H., and Bedard, C. (2006). "Computer Representation to Support Conceptual Structural Design within a Building Architectural Context" Journal of Computing in Civil Engineering 20 76-87.

Mullins, M., Kiregaard, P.H., Jessen, R.Z., and Klitgaard, J. (2005). "A Topology Optimization Approach to Learning in Architectural Design" eCAADe 23 Digital Design: The Quest for New Paradigms, Insituto Superior Tecnico, Lisbon 155-162.

Oxman, R. (1997). Design by re-representation: a model of visual reasoning in design, Design Studies 18 329-347.

Reffat, R.M. (2006). "A Computational System for Enriching Discovery in Architectural Design" in: Kaga, A and Naka, Ryusuke, eds., CAADRIA '06 Proceedings of the 11th Conference on Computer Aided Architectural Design Research in Asia, CAADRIA Morozumi Laboratory, Kumamoto University, 169-177.

Rosenman M.A., Smith G., Ding L., Marchant D., and Maher M.L. (2005). "Multidisciplinary Design in Virtual Worlds", Computer Aided Architectural Design Futures 2005 (Proceedings of the 10th International Conference on Computer Aided Architectural Design Futures / ISBN 1-4020-3460-1) Vienna (Austria) 20–22 June 2005, 433-442.

Sacks, R., Warszawski, A. (1997) "A project model for an automated building system: design and planning phases" Automation in Construction 7 (1997) 21-34.

Sause, R., Martini, K., Powell, G.H. (1992) "Object-Oriented Approaches for Integrated Engineering Design Systems", Journal of Computing in Civil Engineering 6(3) 248-265.

Schenck, Douglas, Wilson, Peter (1994) Information Modeling the EXPRESS way, New York, Oxford, Oxford University Press, ISBN 0-19-508714-3.