WELDED STEEL BEAM DESIGN USING PARTICLE SWARM ANALYSIS

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ABSTRACT: The paper looks at the design of welded steel beams for a typical structure which contains both primary and secondary beams. The underlying theory is fully explained. Also the stress limits and other constraints are defined using practical limits which are defined in Eurocodes. The derived method is only approximate and suitable for early design. In order to give a means of assessing the suitability of PSO for solving the chosen problem, some exact solutions are first calculated using exhaustive search. These are then used to show that a simple PSO approach gives unsatisfactory results and that it is necessary instead to use a layered algorithm.

KEYWORDS: design, welded, steel, beam, particle swarm, constraints.

1 INTRODUCTION

The case for search in design has been well made by those who work in the area. For example Khajehpour and Grierson (2003) estimate that, at the conceptual design stage of a typical 20 storey office block, there are some 167 million feasible design options. Typically designers will look overtly at around 10 options and they will use their expertise to reject further less desirable options but nevertheless, the size of such a search space means that, unless they are very lucky, their design will merely satisfy the constraints. This means they have virtually no chance of finding areas of high performance within the design space. Sadly the design community has not yet accepted this argument.

Various algorithms can be used for design search but generally it is agreed that the best are so called evolutionary algorithms. The choice of which algorithm to use is always one which attracts interest. Wolpert and McCready's (1997) no free lunch theorem states that, it is not possible to find an algorithm which can perform well on all types of problem. This paper looks at the use of Particle Swarm Optimization (PSO) for solving a structural design problem. Examples of the use of PSO for solving Engineering problems are relatively rare, especially in comparison to other evolutionary algorithms such as genetic algorithms, so this is a useful example. Other recent work includes that of Sedlaczek & Eberhard (2006) who demonstrate the ability of PSO in coping with constraints. In this paper, to show how the algorithm performs on the chosen problem domain, it is compared with exact solutions obtained using exhaustive search. The domain is the design of welded steel I beams. Previous work on using evolutionary algorithms for beam design is discussed in Griffiths and Miles (2003). PSO for welded structures has been used in Jalkanen (2006).

The software used has an MS Excel worksheet as its front end and this connects with Visual Basic functions for the algorithms. In the examples given the fabrication costs currently exclude the costs of the joints although work is ongoing in this area. The design is based on criteria given in the relevant Eurocodes.

2 THE PARTICLE SWARM ALGORITHM

The particle swarm algorithm is relatively new (Kennedy and Eberhardt, 1995). As with all other evolutionary algorithms, it works on a population of solutions and requires some form of measure of performance which can be an objective function or can be more loosely defined; what is typically known as a fitness function. If there are n particles within a swarm, each particle has a position x_i, where the subscript i refers to the i'th particle and likewise each particle has a velocity v_i. For each particle within the swarm, its velocity is updated using the formula :-

$$x_{t+1}^{i} = x_{t}^{i} + v_{t+1}^{i} \tag{1}$$

Where the pseudo-velocity v_{t+1}^{i} (it does not have the dimensions of velocity) is calculated using an equation of the form:

$$v_{t+1}^{i} = w_t v_t^{i} + c_1 r_1 (p_t^{i} - x_t^{i}) + c_2 r_2 (g_t - x_t^{i}) \tag{2}$$

In the above the subscript t denotes a pseudo-time step with $t+1$ being the next time step. The w_t parameter is a variable which can be adjusted to determine the rate of convergence. Typically it is set to 1.0 at the start of a run and can then be modified dynamically throughout the process with its value being reduced to aid convergence. The point p_t^i is the best point found so far by individual *i* and g_t is the global best position found to date. The coefficients r_1 and r_2 are random numbers in the range zero to one and typically c_1 and c_2 are given the value of 2, although in some versions of particle swarm they are omitted.

The standard form of a constrained optimisation problem is:

$$\min f(x)$$

$$g_i(x) \le 0 \quad i = 1, \ldots, n$$

Where *f(x)* is the objective function and $g_i(x)$ are constraints. In this work, as is typically done with evolutionary algorithms, the constraints are included in the fitness function with a penalty being used if the constraint is violated as follows :

$$\bar{f}(x) = f(x)\left[1 + \sum_{g_i(x)>0} R g_i(x)\right] \tag{3}$$

R is the penalty parameter which is applied to constraint violations and also may be changed dynamically. Preliminary tests show that the form of the penalty function has a significant impact on the search. If it is too high then the search tends to focus on just a few individuals within the population that manage to avoid the penalty function and this leads to sub-optimal solutions. If it is too low then the search lacks direction and execution times can become excessive. Elitism (i.e. transferring the best individual through to the next generation) has also been found to be an important factor in ensuring a well directed and efficient search.

3 DEFINITION OF BEAM DATA

Consider the welded hybrid beam shown in Figure 1. The notation follows the Finnish national method (Steelbase, 1997) for welded (W) beams with I-profile (I). The yield strengths of the top flange (f_{y1}), the bottom flange (f_{y2}) and the web (f_{yw}) are added to the standard notation.

The problem to be solved is to find the beams with the lowest fabrication costs for the platform structure shown in Figure 2. It is supposed, that the platform is periodically identical in each horizontal direction and only the beams between the four (green) columns need to be considered. The loading is uniform over the entire platform and all the beams are acting as single span beams. Primary beams are from column to column and secondary beams are between the primary beams. Only the ultimate limit state is considered. Deflections are ignored on the basis that if the deflections are too large, then the beams can be pre-cambered.

Initially, the following design criteria are determined both for the primary and secondary beams.

- maximum design bending moment (including load factors) for the principal axis M_{d+} (≥ 0, compression in the top flange),
- minimum design bending moment for the principal axis M_{d-} (≤ 0, compression in the bottom flange),
- maximum absolute design value of shear force for the principal axis $|V_d|$.

Only these actions are considered in this work. It is assumed that the beams are statically determinate and the stiffness of the beams has no effect to the actions.

The input data needed for the design and the cost calculations are

- elastic modulus of steel *E,*
- spacings of lateral supports for the top flange $L_{cr,top}$ and the bottom flange $L_{cr,bot}$,
- imperfection factor α for the top and bottom flange side buckling (approximate theory for the lateral buckling of beams),
- yield strengths f_{y1}, f_{y2} and f_{yw},
- material costs for the top flange k_1, for the bottom flange k_2 and for the web k_w including welding costs (units Euro/m^3),
- painting costs k_p (unit Euro/m^2).

The relevant Eurocodes have been used as will be seen below. The material factor γ_M is 1.0 both for yielding and for buckling at the ultimate limit state.

The standard (EN 1993-1-5, 2004) gives the following limits for the yield strengths

$$f_{y1} \le 2 \cdot f_{yw} \quad , f_{y2} \le 2 \cdot f_{yw}$$

The test cases have been run using the following data :

- spacing of secondary beams 2 m,
- spacing of primary beams 6 m, hence a column spacing of 6 m in both directions,
- uniform dead and live load 5 kN/m^2 for the platform,
- load factor 1.35 for dead load and 1.5 for live load.

The results are as follows in this case

- secondary beam,

  $M_{d+} = 128\ kNm, M_{d-} = 0, |V_d| = 86\ kN, L_{cr,top} = 1\ m$
- primary beam,

  $M_{d+} = 342\ kNm, M_{d-} = 0, |V_d| = 171\ kN, L_{cr,top} = 2\ m$

4 CONSTRAINTS

The constraints for the problem are stated as follows

- the bending stress (top or bottom flange) must be equal to or smaller than the maximum allowed stress,
- the shear stress is equal to or smaller than the allowed stress,
- local buckling is not allowed for the flanges,
- local buckling is allowed for the webs,
- web induced buckling of the flanges is not allowed.

An approximate approach is used, where the flanges alone carry the bending moments and only the web carries the shear force. For the hybrid beams this means that in some cases the web may yield and still resists the shear stresses.

Some approximations are also made for the resistance checks. The vertical position of the action with the respect to the shear center of the beam is not taken into account when considering lateral buckling. Moreover, the maximum value of the bending moment is used for the resistance checks. The distribution of the bending moment along the bar is not taken into account in the analysis. Other approximations used in the resistance checks are described later. Despite these approximations, it is believed that the method described is a valid approach for searching for good solutions at the preliminary design stage.

Using the above theory, the bending moments cause the following axial stresses for the top flange (*index 1*) and for the bottom flange (*index 2*)

$$N_{d1} = \frac{M_d}{h-(\frac{t_1}{2}+\frac{t_2}{2})} \Rightarrow \sigma_1 = \frac{M_d}{[\ h-(\frac{t_1}{2}+\frac{t_2}{2})\ ]\cdot b_1 \cdot t_1} \le \sigma_{1,allowed}$$

$$N_{d2} = \frac{M_d}{h-(\frac{t_1}{2}+\frac{t_2}{2})} \Rightarrow \sigma_2 = \frac{M_d}{[\ h-(\frac{t_1}{2}+\frac{t_2}{2})\ ]\cdot b_2 \cdot t_2} \le \sigma_{2,allowed} \tag{4}$$

These values must be calculated both for the moment M_{d+} ($M_d = M_{d+}$) and for the moment M_{d-} ($M_d = -M_{d-}$).

The allowed axial stresses are (EN 1993-1-1, 2005 and α = 0.49)

$$\sigma_{1,2,allowed} = \frac{\chi_{1,2}\cdot f_{y1,2}}{\gamma_M} = min(1;\frac{1}{\phi_{1,2}+\sqrt{\phi_{1,2}^2-\lambda_{1,2}^2}})\cdot f_{y1,2}$$

$$\phi_{1,2} = 0.5\cdot\left[1+0.49\cdot(\lambda_{1,2}-0.2)+\lambda_{1,2}^2\right]$$

$$\lambda_{1,2} = \sqrt{\frac{b_{1,2}\cdot t_{1,2}\cdot f_{y1,2}\cdot L^2_{cr,top,bot}\cdot 12}{\pi^2\cdot E\cdot b_{1,2}^3\cdot t_{1,2}}} = \frac{L_{cr,top,bot}}{\pi\cdot b_{1,2}}\cdot\sqrt{\frac{12\cdot f_{y1,2}}{E}} \tag{5}$$

Local buckling is not allowed for the flanges and this means (EN 1993-1-1, 2005)

$$\frac{b_{1,2}}{2\cdot t_{1,2}} \le 21\cdot\sqrt{\frac{235}{f_{y1,2}}} \tag{6}$$

The shear force $|V_d|$ causes the following approximative shear stresses to the web

$$\tau = \frac{|V_d|}{h\cdot d} \le \tau_{allowed} \tag{7}$$

The allowed shear stress for the web is (EN 1995-1-5, 2005).

$$If\ 0.346\cdot\frac{h}{d}\cdot\sqrt{\frac{max(f_{y1},f_{y2})}{E}} \le 0.83,\ then\ \tau_{allowed} = \frac{f_{yw}}{\sqrt{3}}$$

$$If\ 0.346\cdot\frac{h}{d}\cdot\sqrt{\frac{max(f_{y1},f_{y2})}{E}} > 0.83,\ then\ \tau_{allowed} = \frac{0.83\cdot f_{yw}}{\sqrt{3}\cdot 0.346\cdot\frac{h}{d}\cdot\sqrt{\frac{max(f_{y1},f_{y2})}{E}}}$$

The last term allows local buckling of the web in shear.

The design criteria for web induced flange buckling of the compression flange is especially important for hybrid beams where thin webs are made of steel with a lower yield stress than that for the flanges. The criteria for the compressed flange is (EN 1993-1-5, 2005) *(1 = top flange, 2 = bottom flange)*:-

$$\frac{h_w}{t_w} \le k\cdot\frac{E}{f_{yf}}\cdot\sqrt{\frac{A_w}{A_{fc}}} \Leftrightarrow (In\ this\ study):$$

$$\frac{h-(t_1+t_2)}{d} \le 0.55\cdot\frac{E}{f_{y1,2}}\cdot\sqrt{\frac{[h-(t_1+t_2)]\cdot d}{b_{1,2}\cdot t_{1,2}}}$$

5 DESIGN SPACE

Six variables are considered during the search
- total height of the beam h,
- thickness of the web d,
- width of the top flange b_1,
- thickness of the top flange t_1,
- width of the bottom flange b_2,
- thickness of the bottom flange t_2.

The design variables have upper and lower limits. Variables can only have integer values meaning that rounding (downwards, during each iteration) is needed. The limits for plate widths varied from 50 to1000 mm, the heights from 80 to 1000 mm, in both cases using 5 mm increments. The allowed plate thicknesses are 4, 5, 6, 8, 10, 12, 15, 16, 20, 22, 25, 30 and 35 mm and allowed yield strengths are 235 and 355 MPa.

6 OBJECTIVE FUNCTION

The objective function consists of material costs (including beam welding) and painting costs as follows :

$$f = k_1\cdot b_1\cdot t_1 + k_2\cdot b_2\cdot t_2 + k_w\cdot(h-t_1-t_2)\cdot d + k_p\cdot[(2\cdot b_1 - d + 2\cdot t_1)+(2\cdot b_2 - d + 2\cdot t_2)+(2\cdot(h-t_1-t_2))] \tag{8}$$

Johansson (2005) is used to estimate the costs for different steel grades. The constraints are as defined above.

7 RESULTS

A routine was written to solve the above by exhaustive search but the execution times were unacceptable. The problem was therefore simplified so that only symmetrical beams were considered, meaning four variables in the design space. Also the search was modified to exclude solutions which had been found to be infeasible in previous generations (e.g. where a dimension exceeded the available space).

Table 1 shows the processing times and the results. The unit costs used are
- 7850 Euro/m^3 (meaning 1 Euro/kg) for S355,
- 7000 Euro/m^3 for S235,
- 2 Euro/m^2 for painting.

Table 1. Results from Initial Exhaustive Search.

Symmetric secondary beam			
Notation	WI405-4-6x160-6x160(355x355x355)	WI450-4-5x170-5x170(355x235x355)	WI490-4-6x190-6x190(235x235x235)
Price in Euro/m	30.30	28.81	32.81
Execution time for exact solution in seconds	0.99	0.92	1.31

Symmetric primary beam			
Notation	WI650-4-8x215-8x215(355x355x355)	WI615-5-8x225-8x225(355x235x355)	WI765-5-8x255-8x255(235x235x235)
Price in Euro/m	51.22	53.47	59.86
Execution time for exact solution in seconds	1.47	1.48	2.00

The execution times were determined using a typical PC and it can be seen that they are relatively small. The design space is of the order of 6 millions options but, by excluding infeasible solutions this can be reduced to around 1% of this, hence leading to the above execution times. By experimentation, it was found that the painting costs had no influence on the solution unless they exceeded 5 Euro/m^2.

From the above, the best secondary beam is a hybrid beam (28.81 Euro/m) and the best primary beam is the beam made totally of S355 steel material.

In table 2 the execution times and the best profiles are shown for a search including unsymmetrical beams.

Table 2. Results for Exhaustive Search and Allowing Unsymmetrical Beams.

	Secondary beam	Primary beam
Notation	WI440-4-6x140-16x55(355x235x355)	WI675-4-8x210-20x75(355x355x355)
Price in Euro/m	28.24	49.10
Execution time for exact solution in seconds	3030	5017

It can be seen, that the execution times are large despite the elimination of the infeasible solutions. The search space for this problem is of the order of 960-1620 million. As can be seen, the price per metre, in comparison with the symmetrical beams, is about 2% lower for the secondary beams and 4% for the primary beams.

8 APPLICATION OF PARTICLE SWARM OPTIMIZATION

The above work gives a benchmark against which the effectiveness of a search process using Particle Swarm Optimization can be compared.

The following parameters were used for the PSO

- Use the lowest feasible values for member one in the initial population (based on the physical limits). This ensures that at least one solution tests the lower bounds;
- The maximum change allowed in any one step for any variable is 30% of its feasible range (i.e. if H has a range between zero and 1000 then the maximum change allowed in H is 300);
- Penalty factor *R*: 0.50;
- Number of particles: 30;
- Inertia *w*: 1.40;
- Individual weight c_1: 2.00;
- Team weight c_2: 2.00;
- Maximum number of iterations: 100.

Initial execution times for single PSOs were about 5 seconds for the unsymmetrical design space. However the best results when compared to the exact solutions were poor, typically 10 % greater and there was a lack of consistency between runs. Therefore a layered PSO was implemented with the PSO parameters being changed during the iterations as follows :

- For the first 10 time steps, the basic PSO without layering is run because it is very fast if inaccurate;
- After 10 time steps the layered PSO is implemented and this is allowed to run for a further 20 time steps;
- The values for the parameters for the layered PSO are:
  - maximum change factor: 0.3 => 0.1
  - penalty factor: 0.50 => 1.00
  - inertia: 1.40 => 0.70
  - individual weight: 2.00 => 1.00
  - team weight: 2.00 => 2.00.

Using lower values for the change parameters as the algorithm converges helps to avoid it “jumping over” the best solution and thus helps to direct the search towards the desired result.

This change increased the execution time to about 40 seconds but the errors were typically well below 10% and in most cases below 5%. It was also found that, when using the layered PSO, very good results were obtained, if the increment for the plate widths was reduced (e.g. to 1 mm) during the iterations with the final result being rounded to the nearest 5 mm. Hence it has been determined that a layered PSO is a better method for this problem.

It must be noted, that the plate thickness lower limit used in this study, 4 mm, is not practical when considering welding beam. Typically 5 mm is the lowest practical thickness for the web. Knowledge such as availability, fabrication restraints etc is something that both the designer and the PSO must have if a truly practical design tool is to be developed.

Figure 3 demonstrates a typical result for a single PSO result for an unsymmetrical primary beam.

The blue profiles in the figure show the shapes of the profile at the limits of the design variables. The black profile is the exact solution and the red one is the PSO solution. From the figures it can be seen how close are the constraints to their limits. The lateral spacing of the bottom flange is 10 m in the calculations. Figure 3 also illustrates the convergence process.

Figure 4 shows a typical result for a layered PSO for the same problem. Note that the scale for the price / iteration graph is different to that in Figure 3.

9 CONCLUSIONS

PSO is a relatively new algorithm and there are relatively few examples of its use for Engineering problems. In this paper a carefully structured experiment is described in which the problem is first solved by exhaustive search. This means that the optimum answer is known and so the performance of the PSO algorithm can be subject to a scientifically valid test.

The results show that a basic PSO does not perform well on the given problem and it is necessary to resort to using a layered PSO which gives much better results. Whether the method can be extended to more complex structural problems is a moot point but at least it would seem that a basic PSO would not be suitable.

Although the findings in this paper are the result of carefully structured tests, it is recognised that these are not exhaustive in terms of the possible permutations of the PSO algorithm and therefore they are only indicative in their nature.

For future work the search should be extended to include features other than fabrication such as design, transportation, erection, use of building, life cycle costs etc. However, the example given demonstrates clearly the need for effective search engines even for simple cases.

REFERENCES

SFS-EN 1993-1-1 (2005). Eurocode 3: Design of steel structures. Part 1-1: General rules and rules for buildings, Finnish Standards Association, Helsinki.

SFS-EN 1993-1-5 (2005). Eurocode 3: Design of steel structures. Part 1-5: Plated structural elements, Finnish Standards Association, Helsinki.

Griffiths, D.R., Miles, J.C. (2003). Determining the optimal cross-section of beams, Advanced Engineering Informatics, 17, 59-76.

Jalkanen J. (2006). Cost optimization of welded steel structures, Proc of IX Finnish Mechanics Days, Lappeenranta University of Technology, Finland.

Johansson B. (2005). Buckling Resistance of Structures of High Strength Steel, in Use and Application of High-Performance Steel for Steel Structures, Structural Engineering Documents 8, IABSE.

Kennedy J., Eberhardt R. (1995). Particle Swarm Optimization, Proc of IEEE Int Conf on Neural Networks, Piscataway NJ, pp. 1942-1948.

Khajehpour, S., Grierson, D. (2003). Profitability versus safety of high-rise office buildings, Journal of Structural and Multidisciplinary Optimization, 25, pp. 1-15.

Sedlaczek, K., Eberhard, P. (2006). Using Augmented Lagrangian Particle Swarm Optimization for Constrained Problems in Engineering, Structural Multidisciplinary Optimization, 32, 277-286.

SteelBase (1997). Finnish Constructional Steelwork Association, Finland, (in Finnish).

Wolpert, D.H., McCready, W.G. (1997). No free lunch theorems for optimisation, IEEE transactions on evolutionary computation, 1(1), 67 – 82.